Math 220
Lecture 17

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Graph Theory

Problems

Happy Ending Problem

There is one fact:

Theorem 0.1. *Any set of five points in the plane in general position has a subset of four points that form the vertices of a convex quadrilateral.*

But how about larger polygons?

Theorem 0.2. *For any positive integer N, any sufficiently large finite set of points in the plane in general position has a subset of N points that form the vertices of a convex polygon.*

But how large it should be?

Three Cottage Problem

This is some problem perhaps you can solve:

> Suppose there are three cottages on a plane and each needs to be connected to the gas, water, and electric companies. Using a third dimension or sending any of the connections through another company or cottage are disallowed. Is there a way to make all nine connections without any of the lines crossing each other?

Knight's Tour

This is a problem concerning with chess:

> A knight is placed on an empty regular chess board which moves according to the chess rule and must visit each square exactly once. Then can we find a path such that this knight will visit every square on the board once and exactly once? Further, can we find a path for this knight such that it ends on a square attacking the square from which it begins? (The latter is called a closed tour, and otherwise it is called an open tour.)

On a regular board, there is one fact, called **Schwenk's Theorem**

Theorem 0.3. *For any $m \times n$ board with m less than or equal to n, a closed knight's tour is always possible unless one or more of these three conditions are true:*

1. *m and n are both odd*
2. *m = 1, 2, or 4; m and n are not both 1*
3. *m = 3 and n = 4, 6, or 8*

Games

Shannon Switching Game

It is a game played on a finite graph assigned with two special vertices A and B which are connected. Each edge of the graph can be either colored or removed. There are two players: "*Short*" and "*Cut*", and they move alternatively. *Cut* is to delete from the graph a non-colored edge of his choice, and *Short* is to color any edge still in the graph. If *Cut* manages to turn the graph into one where A and B are no longer connected, he wins. If *Short* manages to create a colored path from A to B, he wins.

It was invented by Claude Shannon.

Sprouts

It is a game played by two players, starting with a few spots drawn on a sheet of paper. Players take turns, where each turn consists of drawing a line between two spots (or from a spot to itself) and adding a new spot somewhere along the line. The players are constrained by the following rules.

* The line may be straight or curved, but must not touch or cross itself or any other line.

* The new spot cannot be placed on top of one of the endpoints of the new line. Thus the new spot splits the line into two shorter lines.

* No spot may have more than three lines attached to it. For the purposes of this rule, a line from the spot to itself counts as two attached lines and new spots are counted as having two lines already attached to them.

If the one who makes the last move is set to be the winner, this is called a normal play; if he is set to be the loser, this is called a misère play.

This game was invented by John Conway and Michael Paterson at Cambridge University and became a craze there.

Sim

It is a game play on K_6, whose edges are uncolored. Then two players take turns to color the edges: each player can use only one color and each turn he can just color one edge. If a player creates a triangle in his color, he loses immediately.

Philosopher's Football, aka. Phutball

Phutball is played on the intersections of a 19×15 grid using one white stone and as many black stones as needed. In this article the two players are named Ohs (O) and Eks (X). The board is labeled A through P (omitting I) from left to right and 1 to 19 from bottom to top from Ohs' perspective. Rows 0 and 20 represent "off the board" beyond rows 1 and 19 respectively.

Given that specialized phutball boards are hard to come by, the game is usually played on a 19×19 Go board, with a white stone representing the football and black stones representing the men.

The objective is to score goals by using the men (the black stones) to move the football (the white stone) onto or over the opponent's goal line. Ohs tries to move the football to rows 19 or 20 and Eks to rows 1 or 0. At the start of the game the football is placed on the central point, unless one player gives the other a handicap, in which case the ball starts nearer one player's goal.

Players alternate making moves. A move is either to add a man to any vacant point on the board or to move the ball. There is no difference between men played by Ohs and those played by Eks.

The football is moved by a series of jumps over adjacent men. Each jump is to the first vacant point in a straight line horizontally, vertically, or diagonally over one or more men. The jumped men are then removed from the board (before any subsequent jump occurs). This process repeats for as long as there remain men available to be jumped and the player desires. Jumping is optional, there is no requirement to jump. Note that in contrast to checkers, multiple men in a row are jumped and removed as a group.

If the football ends the move on or over the opponent's goal line then a goal has been scored. If the football passes through your goal line, but ends up elsewhere due to further jumps, the game continues.

(Copied directly from Wikipedia, lol)