Math 220 Homework 4

Solutions

Solve the following problems:

1. How many words of length 9 can be formed by 26 English letters?

Solution: Permutation with multiplicities, no restriction: 26^9.

2. How many words of length 9, each of which has 3 f's, 3 t's and 3 g's, can be formed?

Solution: Permutation with speicified multiplicities: $\frac{(3+3+3)!}{3!3!3!}$.

3. There are 3 red beads of 4 black beads. Beads in the same color are regarded to be the same. If we make them in a line, then how many results can happen? (Hint: you may consider using the case of permutation with multiplicities.)

Solution: Permutation with speicified multiplicities: $\frac{(3+4)!}{3!4!}$.

4. In a box there are 30 red beads and 30 black beads. Again, beads in the same color are regarded to be the same. You grab 20 beads, then what is the probability that you get 15 reds and 5 blacks?

Solution: There is only one way to get 15 reds and 5 blacks, because beads in the same color are the same. But to get 20 beads, it is a combination of two elements(red and black) with multiplicities, so there are $\binom{2+20-1}{20}$ ways. And finally, the probability would be $\frac{1}{\binom{2+20-1}{20}}$.

5. As the setting in Problem 4, except that there are 19 reds and 19 blacks, again what is the probability that you get 15 reds and 5 blacks?

Solution: Almost the same argument as in the previous problem, except that you have to exclude two cases: all 20 beads are red and all 20 beads are black. Anyway, the probability would be

$\frac{1}{\binom{2+20-1}{20}-2}$.

6. You have 9 different letters to deliver, and you come to a place where there stand 10 different mailboxes. Then how many way can you drop letters to mailboxes?

Solution: There are 10 ways to deliver for each letter. So totally there are 10^9 ways.

7. How about if those 9 letters are the same (for example, all are written in English)?

Solution: Combination with multiplicities: $\binom{10+9-1}{9}$.

8. There are 8 students to be seated in a row. Suppose Alex does not want to sit in the first place and Zia does not want to sit in the final place, then how many seatings can you arrange?

Solution: No restriction: P_8^8. Alex in the first place: P_7^7.(It is possible Zia is in the final place in those cases.) Zia in the final place: P_7^7.(It is possible Alex is in the first place in those cases.) Alex in the first place and Zia in the final place: P_6^6. So totally we have so many ways: $P_8^8 - P_7^7 - P_7^7 + P_6^6$.

9. Suppose you want to write to 37 different people. Now you have finished writing those 37 letters and addressing 37 envelopes. You close your eyes and randomly stuff one letter into each envelope. What is the probability that just one envelope contains the wrong letter?

Solution: 0, because there is no way you just get exactly one letter wrong.

10. As in Problem 9, what is the probability that precisely two letters are in the wrong envelopes and all others in the correct envelope?

Solution: Two letters get wrong: $\binom{37}{2}$; so the probability would be $\frac{\binom{37}{2}}{P_{37}^{37}}$.