

MATH 217 DIFFERENTIAL EQUATIONS
EXAM 3, JULY 17, 2008
Good Luck!

Problem 0.1 1. Suppose we have

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}.$$

Find its eigenvalues and for each eigenvalue find an associated eigen vector.

2. Use the eigenvalue method to find the general solution to $\frac{d\vec{x}}{dt} = \mathbf{A}\vec{x}$ where $\vec{x} = (x_1, x_2)^T$ is a column vector and \mathbf{A} is the matrix given just as above.

3. Convert that system $\frac{d\vec{x}}{dt} = \mathbf{A}\vec{x}$ above to a second order linear homogeneous equation, say, $Ax'' + Bx' + Cx = 0$. Solve that equation to get the general solution $x_1(t)$ and then the general solution $x_2(t)$.

Problem 0.2 1. Suppose we have

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

Find its eigenvalues and decide the multiplicity for each eigenvalue.

2. Use the eigenvalue method to find the general solution to $\frac{d\vec{x}}{dt} = \mathbf{A}\vec{x}$ where $\vec{x} = (x_1, x_2, x_3)^T$ is a column vector and \mathbf{A} is the matrix given just as above.

Problem 0.3 1. Suppose we have

$$\mathbf{A} = \begin{pmatrix} -5 & -10 & -20 \\ 5 & 5 & 10 \\ 2 & 4 & 9 \end{pmatrix}.$$

Find its eigenvalues and decide the multiplicity for each eigenvalue.

2. Use the eigenvalue method to find the general solution to $\frac{d\vec{x}}{dt} = \mathbf{A}\vec{x}$ where $\vec{x} = (x_1, x_2, x_3)^T$ is a column vector and \mathbf{A} is the matrix given just as above.

Problem 0.4 Prove the following statement: Let $\vec{y}(x)$ be an arbitrary solution to the linear system with constant coefficients $\frac{d\vec{y}(x)}{dx} = \mathbf{A}\vec{y}$. Then $\lim_{x \rightarrow \infty} \vec{y}(x) = \vec{0}$ if and only if every eigenvalue of \mathbf{A} has a negative real part.

Problem 0.5 Find the general solution to the following system:

$$\frac{d}{dx} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}.$$

Perhaps you want to try the eigenvalue method, after taking pains to use that again and again for the previous problems. But I can tell you for this problem there is a much easier way. Just use your brilliance and imagination to crack it! Definitely you will use some knowledge we discussed before.

Problem 0.6 Use the Laplace transform method to find the solution to the initial value problem:

$$y'' - 2y' + y = e^{-t}; y(0) = 0, y'(0) = 1$$

Problem 0.7 (Writing) Summarize what you have learned for the past two weeks. For example, what kind of things we are dealing with, what techniques we are applying, what results we can get, etc.

Problem 0.8 (Bonus 1) If we have a homogeneous high order linear equation $y^{(n)} + a_1y^{(n-1)} + \dots + a_{n-1}y' + a_ny = 0$ with constant coefficients. Convert this equation to a system of linear differential equations. And use the eigenvalue method to deduce the characteristic equation for the homogeneous high order linear equation with constant coefficients. To get full points, you need to show how to calculate the determinant, if any.

Problem 0.9 (Bonus 2) Forget all things about the eigenvalue method, solve this linear system

$$\frac{d}{dt} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & \frac{1}{x} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}.$$

HINT: You may begin with solving for the particular solutions y_2 . Then go back to get y_1 and obtain the solution \vec{y} . Remember this is a system of order 2, so perhaps we need two linearly independent solutions \vec{y}_1 and \vec{y}_2 to form the general solution.