# Math 217 Differential Equations
Final Exam, July 31, 2008
*Good Luck!*

**Problem 0.1** *Find general solutions to the following differential equations.*

1.
$$(1 + ye^{xy})dx + (2y + xe^{xy})dy = 0$$

2. *Use substitution* $u = \sin y$ *to solve*
$$\frac{dy}{dx} = \frac{1}{\cos y} + x \tan y$$

**Problem 0.2** *Find the* ***general*** *solution to this non-homogeneous second order linear differential equation:*

$$y'' + 9y = 2\cos 3x + 3\sin 3x.$$

**Problem 0.3** *Find the general solution to the following system by converting it to a linear second order differential equation:*

$$\begin{cases} \frac{dx}{dt} = x + y \\ \frac{dy}{dt} = 2x - y + e^t. \end{cases}$$

**Problem 0.4** *Find a particular solution to* $y'' + y = \csc^2 x$. *You may use the method of variation of parameters.*

**Problem 0.5** *Find the general solution to the following system:*

$$\frac{d}{dx}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}.$$

**Problem 0.6** *Generally, if a population has a growth rate* $r(t) = k$*, where* $k$ *is a positive constant, then we have a natural growth model* $\frac{dP(t)}{dt} = kP$*.*

*However, birth and death rates of animal populations typically are not constant; instead, they vary periodically with the passage of seasons. Thus we may suppose the growth rate function that varies periodically about its mean value* $k$*:*

$$r(t) = k + b\cos 2\pi t$$

*where* $t$ *is in years and* $k$ *and* $b$ *are positive constants.*

*Now construct a model(a differential equation) for the population* $P(t)$*, and solve that with intial condition* $P(0) = P_0$*.*

*Compare those two models(constant rate vs periodical rate). How would the populations compare after the passage of many years?*

**Problem 0.7 (Writing)** *Pick up any* ***THREE*** *of the following topics and write a short essay for each of them. You should not only list concepts, formulas, but also say something about the ideas, explanations, significance, cautions, or anything you wish to add to make it like a short essay.*

*1. Solution structure of homogeneous and non-homogeneous linear equations(first order, high order(say, n), linear system)*

*2. Linear independence of functions, Wronskians*

*3. Laplace transform*

*4. Power series method*

*5. Euler's method, improved Euler's method, Runge-Kutta method*

**Problem 0.8 (Bonus 1)** *Prove that the functions*

$$f_0(x) \equiv 1, f_1(x) = x, f_2(x) = x^2, \ldots, f_n(x) = x^n$$

*are linearly independent on the real line.*

*Use the result above to prove that for any constant $r$, the functions*

$$g_0(x) = e^{rx}, g_1(x) = xe^{rx}, g_2(x) = x^2e^{rx}, \ldots, g_n(x) = x^ne^{rx}$$

*are linearly independent on the whole real line.*

**Problem 0.9 (Bonus 2)** *Does the result of the second part of the first bonus problem above have something to do with our theories of differential equations? If yes, what is that?*