Monday Exercise
June 16, 2008

Exercise 0.1 *Consider the differential equation*

$$\frac{dy}{dx} + ay = f(x)$$

where $a > 0$ is a constant, and $f(x)$ is a continuous periodic function with 2π as a period.

Find a solution to this equation such that this solution is also a periodic function with 2π as a period.

HINT:

First write a general solution to this equation: $y(x) = \ldots$ Perhaps it will contain an integral. You may set the bounds for the integral with 0 and x.

That $y(x)$ is a continuous periodic function with 2π as a period is equivalent to that $y(x + 2\pi) = y(x)$ for all x. But this won't make work easier. So we will think, if $y(2\pi) = y(0)$ is equivalent to the 2π periodicity, then the rest will be comfortable, because this relation will help us to fix the constant in your original expression of $y(x)$, and then we are done!

So we try to prove that. Generally it is not true. But this time it has something to do with homogeneous/non-homogeneous equations. Perhaps we will have some good tools. One hint is to consider the function $u(x)$ defined to be $u(x) = y(x + 2\pi) - y(x)$, which is a solution to ... and $u(0) = \ldots$

You will use some properties(not only single one!) I listed in class for first order linear homogeneous/non-homogeneous equations.

GOOD LUCK!

Exercise 0.2 *Discuss the methods to solve differential equations with the form*

$$\frac{dy}{dx} = f(\frac{ax + by + c}{mx + ny + l})$$

where a, b, c, m, n and l are constants.

HINT:

If $c = l = 0$, it is a homogeneous equation, so we can use the substituion method $u = \ldots$ to find solutions.

Otherwise, we will discuss in the following two cases:

CASE I: $an - bm \neq 0$

The aim is to convert the original equation to this form via some proper substitution:

$$\frac{dY}{dX} = f(\frac{aX + bY}{mX + nY})$$

How to realize that? Naturally you will think of making some linear substitution. During your endeavor, you will find the condition $an - bm \neq 0$ very important.

CASE II: $an - bm = 0$

This makes us happier because we can make some simplications with that. And this will turn out to be something we have seen. :)

Exercise 0.3 *Read and understand the following applications of differential equations in your textbook:*

Cooling and Heating on Page 40.

Torricelli's Law on Page 41.

Mixture Problem on Page 53.(If you are interested, you may also want to do Problem 40 on Page 57)

In my opinion, the first two are very straightforward. But for the third one, it is very interesting. You can see how to construct a differential equation from the real life. I don't plan to talk about it in details. Maybe if there is enough time, I will show that briefly this week.