INTEGRAL BOUNDS

We know that for indefinite integrals, if we use substitution, we need to return to our original variables. For example:

$$\int \frac{2x\,dx}{x^2+1} \overset{u=x^2+1}{\underset{du=2x\,dx}{=\!=}} \int \frac{du}{u} = \ln|u| + C \overset{u=x^2+1}{=\!=} \ln|x^2+1| + C.$$

Use this way, we can evaluate some definite integrals. For example, use the method of substitution, we can do

$$\int_0^3 \frac{2x\,dx}{x^2+1} = \ln|x^2+1|\Big|_0^3 = \ln 10.$$

However, if we use the method of substitution to evaluate definite integrals, we do need to go back to the original variables so long as we can keep the track of bounds for the new variables. For the example above, we can do like this:

$$\int_0^3 \frac{2x\,dx}{x^2+1} \overset{u=x^2+1}{=\!=} \int_{0^2+1}^{3^2+1} \frac{du}{u} = \ln|u|\Big|_1^{10} = \ln 10$$

Note that here the bounds are for $u=x^2+1$, so the lower and upper bounds will change. We just need to plug in the values of x to fix the values of u which will be the new bounds.

Remark: 1. This makes you writing and computing a little easier.

2. Actually if you write $\int_0^3 \frac{du}{u}$ for that problem, it is incorrect because others may assume that bounds for u are 0 and 3.