A Counterexample
To the Mean Value Theorem for Integrals
Without Continuity Condition

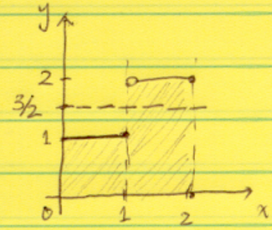
In class we saw the MVT as follows: If f is continuous on $[a,b]$, then $\exists\, c \in [a,b]$ s.t. $\int_a^b f(x)dx = f(c)\cdot(b-a)$.

If we are not given the continuity condition, even if it's still integrable, the conclusion may fail. Here is one counterexample.

Define $f(x)$ on $[0,2]$ by $f(x) = \begin{cases} 1 & 0 \le x \le 1 \\ 2 & 1 < x \le 2 \end{cases}$

This function is still integrable:

$$\int_0^2 f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx$$
$$= \int_0^1 1\cdot dx + \int_1^2 2\cdot dx$$
$$= 1 + 2 = 3$$



But it is NOT continuous on $[0,2]$, as it's not continuous at $x=1$.

If we calculate $\frac{1}{b-a}\int_a^b f(x)dx = \frac{1}{2-0}\int_0^2 f(x)dx = \frac{3}{2}$, we can see that there is NO $c \in [0,2]$ s.t. $f(c) = \frac{3}{2}$.

NOTE: If by any chance I mentioned in class with "integrable condition" instead of "continuous condition" for this theorem, please correct it.