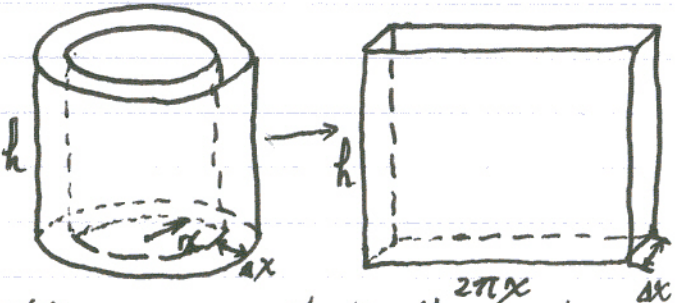
Shell-Method

In class we see that if we want to find the volume of the sphere by applying the shell method, we need to find the volume of that solid generated by revolving the rectangle.

In class we used the direct way to find the volume by dropping $(\Delta x)^2$ term. Here is another approximation.



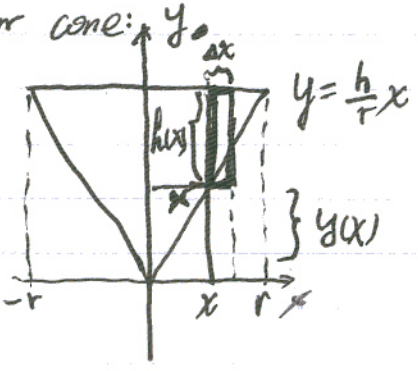
By the picture on the right, we see that the volume is approximately $2\pi x \cdot h \cdot \Delta x$

For the problem of the circular cone:

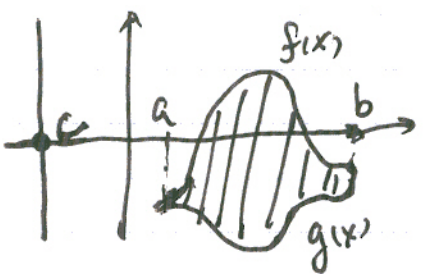
If we revolve the rectangle about y-axis, the the volume will be $V_x = 2\pi \cdot x \cdot h(x) \cdot \Delta x$, and $h(x) = h - y(x) = h - \frac{h}{r}x$.

So the volume of the cone is

$$V = \int_0^r 2\pi x \cdot (h - \frac{h}{r}x)\, dx$$



And now you can generalize this idea for other functions. For example, find the volume of the solid generated by revolving the shaded area about the line $x = c$ $(a > c)$



$$V = \int_a^b 2\pi \cdot (x - c)\,(f(x) - g(x))\, dx$$