

$$\begin{aligned}
& \int_0^{\pi} \sin^4 t \cos^2 t \, dt \\
&= \int_0^{\pi} (\sin^2 t)^2 \left(\frac{1 + \cos 2t}{2} \right) dt \\
&= \int_0^{\pi} \left(\frac{1 - \cos 2t}{2} \right)^2 \left(\frac{1 + \cos 2t}{2} \right) dt \\
&= \frac{1}{8} \int_0^{\pi} (1 - \cos 2t)(1 - \cos^2 2t) dt \\
&= \frac{1}{8} \int_0^{\pi} (1 - \cos 2t - \cos^2 2t + \cos^3 2t) dt \\
&= \frac{1}{8} \int_0^{\pi} \left(1 - \cos 2t - \frac{1 + \cos 4t}{2} + \cos^3 2t \right) dt \\
&= \frac{1}{8} \int_0^{\pi} \left(\frac{1}{2} - \cos 2t - \frac{1}{2} \cos 4t + \cos^2 2t \cdot \cos 2t \right) dt \\
&= \frac{1}{8} \left[\int_0^{\pi} \frac{1}{2} dt - \int_0^{\pi} \cos 2t \, dt - \frac{1}{2} \int_0^{\pi} \cos 4t \, dt + \int_0^{\pi} (1 - \sin^2 2t) \cos 2t \, dt \right] \\
&= \frac{1}{8} \left[\frac{\pi}{2} + \frac{1}{2} \int_0^{\pi} (1 - \sin^2 2t) \cos 2t \, d(2t) \right] \\
&= \frac{1}{8} \left[\frac{\pi}{2} + \frac{1}{2} \int_0^{\pi} (1 - \sin^2 2t) d(\sin 2t) \right] \\
&= \frac{1}{8} \left[\frac{\pi}{2} + \frac{1}{2} \left(\sin 2t \Big|_0^{\pi} - \frac{1}{3} (\sin 2t)^3 \Big|_0^{\pi} \right) \right] \rightarrow \text{or use substitution} \\
&= \frac{1}{8} \left(\frac{\pi}{2} + 0 \right) = \frac{\pi}{16} \quad \text{Yeah!} \quad \begin{array}{l} u = \sin 2t \Rightarrow \\ \int_0^0 (1 - u^2) du = 0 \end{array}
\end{aligned}$$