Quiz 9, Math 128
Calculus II for the Life, Social and Managerial Sciences

Name: KEY

Question 1. Fill in the blanks.

(a) A solution of a differential equation is any function $f(t)$ such that the differential equation becomes a true statement when y is replaced by $f(t)$, y' by $f'(t)$, and so forth.

(b) $y' + a(t)y = b(t)$ is a first order linear differential equation in standard form.

Question 2. Calculations.

(a) Is $f(t) = (e^{-t}+1)^{-1}$ a solution of $y' + y^2 = y$? Justify your answer.

let $y = f$. The $y' = -1(e^{-t}+1)^{-2}(-e^{-t})$

So $y' + y^2 = e^{-t}(e^{-t}+1)^{-2} + [(e^{-t}+1)^{-1}]^2 = (e^{-t}+1)^{-2}(e^{-t}+1)$

$= (e^{-t}+1)^{-1}$

$= y$

$\therefore$ $y' + y^2 = y$, so $\boxed{\text{yes, it is a solution.}}$

(b) Solve the differential equation with the given initial conditions.
$ty' + y = \ln t$, $y(e) = 0$, $t > 0$.

divide by $t \Rightarrow y' + \frac{y}{t} = \frac{\ln t}{t}$

$a(t) = \frac{1}{t} \Rightarrow A(t) = \ln t \Rightarrow e^{A(t)} = e^{\ln t} = t$

multiply by $t \Rightarrow (yt)' = \ln t$

integrate $\Rightarrow yt = \int \ln t \, dt$

$= t\ln t - t + C$

So $yt = t\ln t - t + C$.

$\Rightarrow y = \ln t - 1 + \frac{C}{t}$

Since $y(e) = 0$, $0 = \ln(e) - 1 + \frac{C}{e} = 1 - 1 + \frac{C}{e} = \frac{C}{e}$

So $C = 0$

$\Rightarrow \boxed{y = \ln t - 1}$