Math 3333 — Intermediate Analysis
Midterm 2

1. Which of the following subsets of $\mathbf{R}$ are connected?

(a) $\mathbf{Z}$, the integers

(b) $\mathbf{Q}$, the rational numbers

(c) $[0, \infty)$

(d) $(0, \infty)$

(e) $(-\infty, 0) \cup (0, \infty)$

2. Let (t_n) be a convergent sequence of real numbers and let $t = \lim t_n$. Prove that the set $\{t_1, t_2, \ldots\} \cup \{t\}$ is compact.

3. Give examples meeting each of the following descriptions. In each case include a brief explanation of why your example has the stated condition(s).

(a) a Cauchy sequence of real numbers

(b) a sequence in $[0, 1]$ that is **not** Cauchy

(c) a sequence (s_n) of real numbers which is not Cauchy, but which satisfies $|s_{n+1} - s_n| \to 0$.

4. Let $f : \mathbf{R} \to \mathbf{R}$ be the function

$$f(t) = \begin{cases} t + 1 & \text{if } t < 0 \\ t & \text{if } t \geq 0 \end{cases}.$$

(a) Find an open set $U \subseteq \mathbf{R}$ such that $f^{-1}(U)$ is not open.

(b) Find a convergent sequence (t_n) of real numbers such that $f(t_n) \not\to f(t)$ where $t = \lim t_n$.

5. Let $f : [0, 1] \to \mathbf{R}$ be a continuous function. Suppose $f(t)$ is an integer for every value of t and that $f(0) = 6$. Prove that $f(1) = 6$.