1. Which of the following subsets of $\mathbb{R}$ are connected?
   (a) $\mathbb{Z}$, the integers
   (b) $\mathbb{Q}$, the rational numbers
   (c) $[0, \infty)$
   (d) $(0, \infty)$
   (e) $(-\infty, 0) \cup (0, \infty)$

2. Let $(t_n)$ be a convergent sequence of real numbers and let $t = \lim t_n$. Prove that the set $\{t_1, t_2, \ldots\} \cup \{t\}$ is compact.

3. Give examples meeting each of the following descriptions. In each case include a brief explanation of why your example has the stated condition(s).
   (a) a Cauchy sequence of real numbers
   (b) a sequence in $[0, 1]$ that is not Cauchy
   (c) a sequence $(s_n)$ of real numbers which is not Cauchy, but which satisfies $|s_{n+1} - s_n| \to 0$.

4. Let $f : \mathbb{R} \to \mathbb{R}$ be the function
   \[ f(t) = \begin{cases} 
   t + 1 & \text{if } t < 0 \\
   t & \text{if } t \geq 0
   \end{cases} \]
   (a) Find an open set $U \subseteq \mathbb{R}$ such that $f^{-1}(U)$ is not open.
   (b) Find a convergent sequence $(t_n)$ of real numbers such that $f(t_n) \not\to f(t)$ where $t = \lim t_n$.

5. Let $f : [0, 1] \to \mathbb{R}$ be a continuous function. Suppose $f(t)$ is an integer for every value of $t$ and that $f(0) = 6$. Prove that $f(1) = 6$. 