

Please follow directions carefully and show all your work. Show your work and write your answers on the lined paper or graph paper, not on this sheet (except for problem 12). There is a total of 65 points on the exam.

Part I. (60 points) You may not use a graphing calculator on this part of the exam.

(2) 1. Suppose f is a function which has an inverse function f^{-1} . Then most of the following statements are sure to be true. Which one statement may be false?

(A) f is one-to-one.

(B) The domain of f^{-1} is the range of f .

(C) $f^{-1}(x) = \frac{1}{f(x)}$

(D) If $f(2) = 3$, then $f^{-1}(3) = 2$.

(E) For any x in the domain of f , $f^{-1}(f(x)) = x$.

(F) The graphs of f and f^{-1} are mirror images across the line $y = x$

(2) 2. Suppose f is a polynomial function with degree n . Then most of the following statements are sure to be true. Which one statement may be false?

(A) f is continuous.

(B) The domain of f is all real numbers.

(C) The range of f is all real numbers.

(D) f has at most n x -intercepts.

(E) f has at most $n-1$ extreme points (maximum or minimum points).

(F) f has no vertical asymptotes.

(2) 3. Suppose f is an exponential function. Specifically, suppose $f(x) = a^x$ with $a > 0$ and $a \neq 1$. Then most of the following statements are sure to be true. Which one statement may be false?

(A) f is continuous.

(B) The domain of f is all real numbers.

(C) $f(1) = a$

(D) If $a > 1$, then f is an increasing function.

(E) The x -axis is a horizontal asymptote of the graph of f

(F) f grows more slowly than the function $g(x) = x^5$.

(2) 4. Suppose f is a logarithmic function. Specifically, suppose $f(x) = \log_a x$ with $a > 0$ and $a \neq 1$. Then most of the following statements are sure to be true. Which one statement may be false?

- (A) f is continuous.
- (B) The domain of f is all real numbers.
- (C) $f(1) = 0$
- (D) $f(a) = 1$
- (E) The y -axis is a vertical asymptote of the graph of f .
- (F) f grows more slowly than the function $g(x) = x^5$.

(2) 5. Most of the following statements about the six trigonometric functions are true. Which one statement is false?

- (A) All of the trigonometric functions are continuous.
- (B) All of the trigonometric functions are periodic.
- (C) All of the trigonometric functions fail to be one-to-one.
- (D) All of the trigonometric functions are positive for angles with terminal side in quadrant I.
- (E) All of the trigonometric functions are symmetric with respect to either the y -axis or the origin.
- (F) For any angle, $\sin^2\theta + \cos^2\theta = 1$

6. Find the value of $16^{\frac{3}{4}}$.

7. Simplify, and write your final answer with no negative exponents.

$$\frac{(xy^3)^{-4}}{y^{-3}}$$

(7) 8. Solve for x , and write your final answer in interval notation

$$2x^2 < x + 10$$

9 (a) Find the distance between the points $(4, -1)$ and $(-2, 7)$

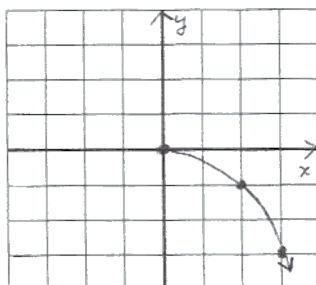
(b) Find the equation of the line passing through the points $(4, -1)$ and $(-2, 7)$. You do not need to simplify your answer.

(5) 10. Carefully graph the piecewise-defined function $f(x) = \begin{cases} -2x+3 & \text{if } x < 2 \\ -3 & \text{if } x \geq 2 \end{cases}$

Be sure to mark a scale on each axis.

(2) 11. Let $f(x) = x^2 - 3x$ and let $g(x) = 5x + 2$. Find $(f \circ g)(x)$. You do not need to simplify.

(1) 12. Suppose the graph of a function $y = f(x)$ is as shown below. Draw the graph of $y = f(-x)$. You may draw it on the same coordinate plane, if you wish.



(5) 13. Let $f(x) = \frac{3x-1}{x+2}$. This function has exactly one y -intercept, one x -intercept, one vertical asymptote, and one horizontal asymptote.

- (a) Find the y -intercept. Write it as an ordered pair.
- (b) Find the x -intercept. Write it as an ordered pair.
- (c) Find the vertical asymptote. Write it as the equation of a line.
- (d) Find the horizontal asymptote. Write it as the equation of a line.

14. Find the value of $\log_3\left(\frac{1}{3}\right)$.

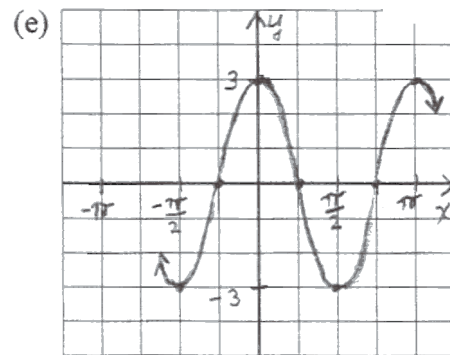
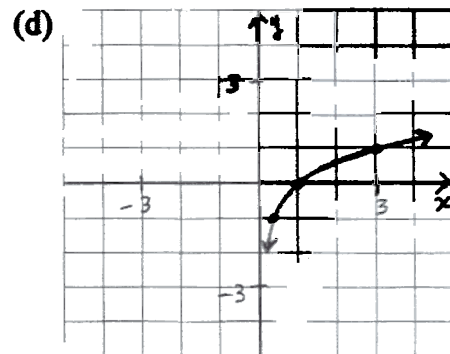
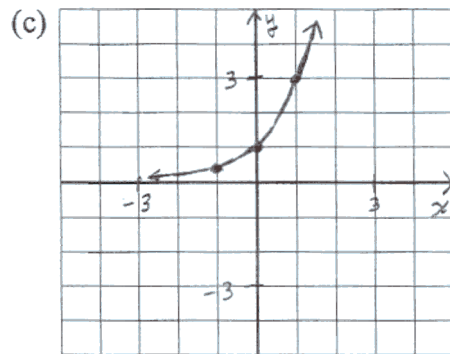
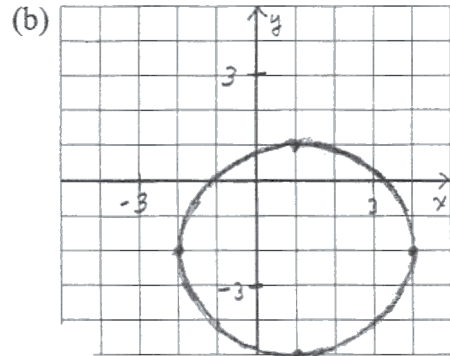
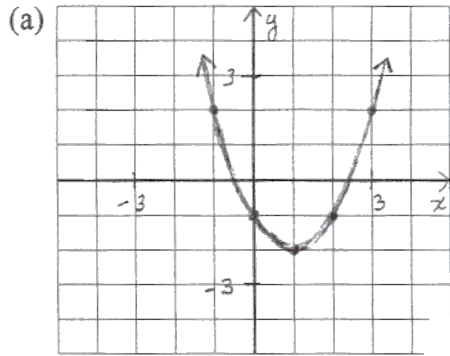
15. Find the value of each of the following.

- (a) $\cos(-120^\circ)$
- (b) $\sin\left(\frac{3\pi}{2}\right)$
- (c) $\tan^{-1}(1)$

(3) 16. We know that $\tan x = \frac{\sin x}{\cos x}$. Write the other three trigonometric functions in terms of $\sin x$ and/or $\cos x$:

- (a) $\cot x =$
- (b) $\sec x =$
- (c) $\csc x =$

(10) 17. Write the equation corresponding to each of the following graphs.



Part II. (5 points) You may use a graphing calculator on this part of the exam

(5) 18. What interest rate (to the nearest percent) is needed for \$1000 to grow to \$8000 in 16 years, if interest is compounded continuously?

Foundations for Calculus Fall 2002 Final Exam Solutions

c

2 c

3 F

4 B

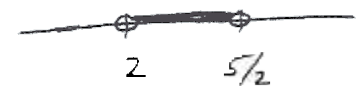
5 A

6 $6^{3/4}$ $(\sqrt[4]{16})^3$ 2^3 8

7 $\frac{(xy^3)^{-4}}{y}$ $\frac{x^{-4}y^{-12}}{y^3}$ $x^{-4}y^{-9}$ $x^{-4}y^{-9}$

8 $2x^2 < x + 0$

to find partition numbers $2x$ $x + 10$

$2x$	x	10	0	test	F	
$(2x$	$5x$	x	$2)$	0	test 0	
$2x$	5	0	$x+2$	0	test 3	18 13 F
x	$5/2$	x	2	2 and 5/2 are not solutions		

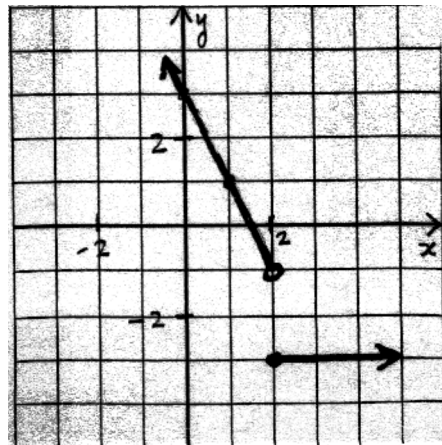
answer $(-2, 5/2)$

9 (a) d $\sqrt{(-2-4)^2 + (7-(-1))^2}$ $\sqrt{36+64}$ $\sqrt{100}$ 10

(b) m $\frac{7-(-1)}{2-4}$ $\frac{8}{-6}$ $\frac{4}{3}$

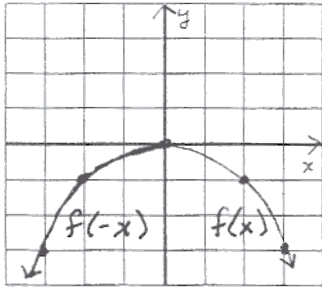
$y + \frac{4}{3}(x - 4)$

$f(x) \begin{cases} 2x & \text{if } x < 3 \\ 3 & \text{if } x \geq 3 \end{cases}$



$$f(x) = 3x \quad g(x) = 5x + 2$$

$$(f \circ g)(x) = f(g(x)) = f(5x + 2) = (5x + 2)^2 \cdot 3(5x + 2)$$



(a) $f(x) = 3x^2$

(b) $f(x) = 3x^2$ at $x = 0$ and $x = \frac{1}{3}$

(c) $f(x) = 3x^2$ at $x = 0$ and $x = 2$

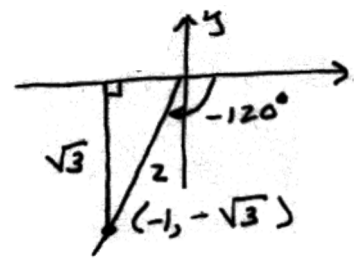
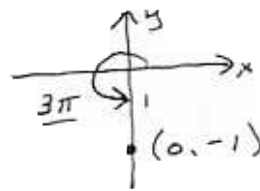
(d) $f(x) = 3x^2$ at $x = \frac{1}{2}$ and $x = 3$

$$\log_3(\quad) = 1$$

(a) $\cos(20^\circ) = \frac{x}{2}$

(b) $\sin(3\pi) = \frac{y}{r}$

(c) $\tan(45^\circ) = \frac{\pi}{4}$



(a) $\sec x = \frac{r}{x}$

(b) $\sec x = \frac{1}{\cos x}$

(c) $\sec x = \frac{1}{\cos x}$

$$7 \quad (a) \quad y \quad (x \quad 2$$

$$(b) \quad (x \quad + \quad (y \quad 2 \quad 9$$

$$(c) \quad y$$

$$(d) \quad y \quad \log x$$

$$(e) \quad y \quad 3a \quad 2x$$

A Per t

$$80 = 0 \quad 000 e^{rt}$$

$$8 \quad e^{6r}$$

$$\ln \quad 6r$$

$$r \quad \frac{\ln 8}{16} \approx 1.9965$$

37