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This exam has 17 questions:

- 15 multiple choice worth 6 points each.
- 2 hand graded worth 25 points total.

Important:

- No graphing calculators!
- For the multiple choice questions, mark your answer on the answer card.
- Show all your work for the written problems. You will be graded on the ease of reading your solution.
- You are allowed a 3×5 note card for the exam.

1. What is the domain of the function $f(x) = \frac{1}{\sqrt{x+1}}$?

   (a) $(-\infty, -1)$
   (b) $(-\infty, -1]$
   (c) $(-\infty, 1)$
   (d) $(-\infty, 1]$
   (e) $(-\infty, -1) \cup (-1, \infty)$
   (f) $(-1, 1)$
   (g) $(-1, \infty)$
   (h) $[-1, \infty)$
   (i) $(-1, 1)$
   (j) $(1, \infty)$
   (k) $(-\infty, \infty)$

2. Suppose $f(x) = x^4 + 2$, $g(x) = \frac{1}{\sqrt{x}}$, and $h(x) = x^2$. What is $(f \circ g \circ h)(x)$?

   (a) $\left(\frac{1}{x^2} + 2\right)^2$
   (b) $\frac{1}{\sqrt{x^8+2}}$
   (c) $\frac{1}{x^4} + 2$
   (d) $x^{11/2} + 2x^{3/2}$
   (e) $\frac{1}{x^4+2}$

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3. Simplify the expression $\frac{x^3y^{-2}z}{x^{-3}yz^3}$.

   (a) $\frac{x^6y^3}{z^2}$
   (b) $\frac{x^6}{yz^2}$
   (c) $\frac{y^3}{z^2}$
   (d) $\frac{1}{x^6y^3z^2}$
   (e) $\frac{x^6y}{z^2}$
   (f) $\frac{x^6}{y^3z^2}$
   (g) $\frac{1}{yz^2}$
   (h) $\frac{1}{x^6yz^2}$
   (i) $\frac{y}{x^6z^2}$

4. Suppose that you put 100 dollars into a bond that pays 12% annual interest, compounded monthly. How much money will be in the account after 10 years? (Pick the closest amount)

   (a) \$110
   (b) \$113
   (c) \$220
   (d) \$259
   (e) \$314
   (f) \$330
   (g) \$9,270,907

5. Find the limit:

$$\lim_{x\to\infty} \frac{2x + 5x^2 - x^3}{4 + x - 5x^2 + 2x^3}$$

   (a) -2
   (b) $\frac{-1}{2}$
   (c) $\frac{-1}{4}$
   (d) $\frac{-1}{5}$
   (e) $\frac{1}{2}$
   (f) 0
   (g) 1
   (h) 2
   (i) 5
   (j) DNE

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6. Find the limit:

$$\lim_{x \to 3} \frac{\sqrt{x+1}-2}{x-3}$$

(a) -2
(b) 0
(c) $\frac{1}{4}$
(d) $\frac{1}{3}$
(e) $\frac{3}{2}$
(f) 2
(g) 3
(h) DNE

7. Find the limit:

$$\lim_{x \to 3} \frac{3+2x^2}{x(x-2)}$$

(a) 0
(b) $\frac{3}{4}$
(c) 1
(d) $\frac{5}{3}$
(e) 2
(f) 3
(g) $\frac{18}{5}$
(h) 7
(i) DNE

8. Find the limit:

$$\lim_{x \to 1} \frac{4x}{2x-2}$$

(a) -2
(b) -1
(c) $\frac{-1}{2}$
(d) 0
(e) $\frac{1}{2}$
(f) 1
(g) 2
(h) 4
(i) DNE

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9. What is the equation of the line that passes through the points $(6, 1)$ and $(-2, -3)$?

   (a) $y = \frac{-1}{2}x + 4$
   (b) $y = 2x - 11$
   (c) $y = 6x + \frac{2}{3}$
   (d) $y = 3x + 1$
   (e) $y = \frac{1}{3}x + \frac{1}{3}$
   (f) $y = \frac{1}{2}x - 2$
   (g) $y = \frac{1}{3}x - \frac{8}{3}$
   (h) $y = 2x + 1$

10. Find the equation of a line that is parallel to the line $y = x/2$ and tangent to the curve $y = \sqrt{x}$ at some point.

   (a) $y = x/2 + 1/2$
   (b) $y = x$
   (c) $y = 2x$
   (d) $y = 2x + 1$
   (e) $y = x/2 - 1/2$
   (f) $y = x/2$
   (g) $y = x/2 + 1$
   (h) $y = x/2 - 1$
   (i) $y = -2x - 1/2$

11. What is the derivative of the function $y = \frac{1}{\sqrt{2x^2+1}}$?

   (a) $-\frac{4x}{\sqrt{2x^2+1}}$
   (b) $2x\sqrt{2x^2+1}$
   (c) $-\frac{2x}{(2x^2+1)^{3/2}}$
   (d) $-\frac{2x}{\sqrt{2x^2+1}}$
   (e) $4x\sqrt{2x^2+1}$
   (f) $\sqrt{2x^2+1}$
   (g) $-\frac{4x}{(2x^2+1)^{3/2}}$
   (h) $-\frac{1}{(2x^2+1)^{3/2}}$
   (i) $-\frac{1}{2(2x^2+1)^{3/2}}$

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12. Which of the following functions are differentiable at $x = 0$?

I. $f(x) = x^2$

II. $f(x) = |x|$

III. $f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

IV. $f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

(a) Only I
(b) Only I and II
(c) Only I and III
(d) Only I and IV
(e) Only III and IV
(f) Only I, II and III
(g) Only I, III, and IV
(h) Only I, II, and IV
(i) All of them
(j) None of them

13. Find the equation of the tangent line to the curve $y = x(x+1)(x-1)$ at the point $(1, 0)$.

(a) $y = 3x$
(b) $y = 0$
(c) $y = 2x + 1$
(d) $y = 2x - 2$
(e) $y = -x + 1$
(f) $y = x$
(g) $y = x - 1$
(h) $y = 2x$
(i) $y = 3x - 3$
(j) $y = 3x + 1$

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14. The typical energy consumption during one day for an office building is approximately given by $E(t) = -t(36t+1)(t-36)$, where t (between 0 and 24) is the number of hours since midnight, and $E(t)$ is the number of kilowatt-hours consumed since midnight. What is the rate of consumption (in kilowatts) at noon?

    (a) 36 kW
    (b) 108 kW
    (c) 1295 kW
    (d) 2518 kW
    (e) 9086 kW
    (f) 10380 kW
    (g) 13201 kW
    (h) 15564 kW
    (i) 124704 kW
    (j) 249120 kW

15. For the same office building as in the previous problem, what the average rate of consuption over the entire day (from midnight to the following midnight)?

    (a) 36 kW
    (b) 108 kW
    (c) 1295 kW
    (d) 2518 kW
    (e) 9086 kW
    (f) 10380 kW
    (g) 13201 kW
    (h) 15564 kW
    (i) 124704 kW
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WRITTEN PROBLEM—SHOW YOUR WORK

16. (a) (5 pts) Use the power rule to compute the derivative of $\frac{1}{\sqrt{x}}$.

(b) (10 pts) Find the derivative of $\frac{1}{\sqrt{x}}$ again, this time using the limit definition of the derivative.

17. (10 pts) Write 1-3 sentences that describe the relationship between differentiability and continuity.