MIDTERM 1 MATH 127 Fall 2006

Name: Solutions

Instructions: You have 2 hours to take this exam. You may use one 3×5 notecard. Calculators are **not** allowed.

Show your work and simplify your answers.

1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
7	/10
8	/10
9	/10
10	/10
Total	/100

1. (a) Determine the slope, x-intercept, and y-intercept of the line $3x + 4y = 5$.

x-intercept: $3x = 5 \rightarrow \boxed{x = 5/3}$

y-intercept: $4y = 5 \rightarrow \boxed{y = 5/4}$

slope: $4y = -3x + 5$

$$y = \frac{-3}{4}x + \frac{5}{4} \rightarrow \boxed{m = \frac{-3}{4}}$$

(b) Find the equation of the line through the points $(2, 5)$ and $(6, 3)$.

$$m = \frac{3-5}{6-2} = \frac{-2}{4} = \frac{-1}{2}$$

$$y - 5 = -\frac{1}{2}(x-2)$$

$$\boxed{y = -\frac{1}{2}x + 6}$$

2. Complete the square and find the vertices and roots of the following quadratic polynomials:

(a) $r^2 - 4r + 5$

$(r^2 - 4r + 4) - 4 + 5$

$\boxed{(r-2)^2 + 1}$

$\boxed{\text{vertex } (2,1)}$

roots: $(r-2)^2 + 1 = 0$

$(r-2)^2 = -1$

$\boxed{\text{no real roots}}$

(b) $3x^2 + 18x + 20$

$3(x^2 + 6x + 9) + 20 - 27$

$\boxed{3(x+3)^2 - 7}$

$\boxed{\text{vertex } (-3,-7)}$

roots: $3(x+3)^2 - 7 = 0$

$3(x+3)^2 = 7$

$(x+3)^2 = 7/3$

$x + 3 = \pm\sqrt{7/3}$

$\boxed{x = -3 \pm \sqrt{7/3}}$

3. Simplify the expressions in parts (a), (b), and (c). Rationalize the denominator in parts (d) and (e).

(a) $(xy)^2 \cdot (x/y)^4$ $= x^2y^2 \cdot x^4/y^4 = \boxed{x^6/y^2}$

(b) $(z^3)^{-4}$ $= \boxed{z^{-12}}$

(c) $\sqrt[3]{a}/\sqrt{a}$ $= a^{1/3} \cdot a^{-1/2} = a^{1/3 - 1/2} = \boxed{a^{-1/6}}$

(d) $\frac{2-\sqrt{3}}{2+\sqrt{3}}$ $\quad \frac{2-\sqrt{3}}{2+\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{4-4\sqrt{3}+3}{4-3} = \boxed{7-4\sqrt{3}}$

(e) $\frac{1}{2-\sqrt[3]{7}}$ $\quad \frac{1}{2-7^{1/3}} \cdot \frac{4+2\cdot 7^{1/3}+7^{2/3}}{4+2\cdot 7^{1/3}+7^{2/3}} = \frac{4+2\cdot 7^{1/3}+7^{2/3}}{8-7}$

$= \boxed{4+2\cdot 7^{1/3}+7^{2/3}}$

4. Evaluate the following limits:

(a) $\lim_{x\to 2} \frac{2x^2+x-10}{x-2}$

$$\lim_{x\to 2} \frac{2x^2+x-10}{x-2} = \lim_{x\to 2} \frac{(2x+5)\cancel{(x-2)}}{\cancel{x-2}} = \boxed{9}$$

(b) $\lim_{x\to 3} \frac{4x-12}{3x-9}$

$$\lim_{x\to 3} \frac{4x-12}{3x-9} = \lim_{x\to 3} \frac{4\cancel{(x-3)}}{3\cancel{(x-3)}} = \boxed{\frac{4}{3}}$$

(c) $\lim_{x\to 1} \frac{x^2-2x+1}{\sqrt{x}-1}$

$$\lim_{x\to 1} \frac{x^2-2x+1}{\sqrt{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \lim_{x\to 1} \frac{(x-1)^{\cancel{2}}(\sqrt{x}+1)}{\cancel{x-1}} = \boxed{0}$$

5. The height of a rocket is given by the formula $H(t) = -5t^2 + 20t + 60$, with H in meters and t in seconds.

(a) Find the average velocity of the rocket between $t = 1$ and $t = 3$ seconds; between $t = 1$ and $t = 2$ seconds; and between $t = 0$ and $t = 2$ seconds.

t	$H(t)$
0	60
1	75
2	80
3	75

$t=1$ to $t=3$: $\frac{75-75}{2} = \boxed{0 \text{ m/sec}}$

$t=1$ to $t=2$: $\frac{80-75}{1} = \boxed{5 \text{ m/sec}}$

$t=0$ to $t=2$: $\frac{80-60}{2} = \boxed{10 \text{ m/sec}}$

(b) Find its average velocity between $t = 1$ and $t = 1 + h$ seconds, for arbitrary h.

$$\frac{(-5(1+h)^2 + 20(1+h) + 60) - 75}{h} = \frac{\cancel{-5} - 10h - 5h^2 \cancel{+20} + 20h \cancel{+60} \cancel{-75}}{h}$$

$$= \frac{-5h^2 + 10h}{h}$$

$$= \boxed{-5h + 10 \text{ m/sec}}$$

(c) Find its instantaneous velocity at time $t = 1$.

$$\lim_{h \to 0} (-5h + 10) = \boxed{10 \text{ m/sec}}$$

6. (continuation of problem 5)

(a) Find its average velocity between time t and $t+h$ seconds, for arbitrary t and h.

$$\frac{(-5(t+h)^2+20(t+h)+60)-(-5t^2+20t+60)}{h} = \frac{\cancel{-5t^2}-10th-5h^2+\cancel{20t}+20h+\cancel{60}+\cancel{5t^2}-\cancel{20t}-\cancel{60}}{h}$$

$$= \frac{-10th-5h^2+20h}{h}$$

$$= \boxed{-10t-5h+20 \ \text{m/sec}}$$

(b) Find its instantaneous velocity at time t.

$$\lim_{h\to 0}(-10t-5h+20) = \boxed{-10t+20 \ \text{m/sec}}$$

7. Using the limit definition of the slope of the tangent line, find the slopes of the lines tangent to each of the following functions at the given points. Find the equation of the tangent line to the function at each point.

(a) $y = x^2 + x$ at $(2, 6)$

$$\lim_{x\to 2} \frac{(x^2+x)-6}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+3)}{\cancel{x-2}} = \boxed{5}$$

$$y-6=5(x-2) \rightarrow \boxed{y=5x-4}$$

(b) $y = \sqrt{x-2}$ at $(6, 2)$

$$\lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6} \cdot \frac{\sqrt{x-2}+2}{\sqrt{x-2}+2} = \lim_{x\to 6} \frac{(x-2)-4}{(x-6)(\sqrt{x-2}+2)} = \lim_{x\to 6} \frac{\cancel{x-6}}{\cancel{(x-6)}(\sqrt{x-2}+2)}$$

$$= \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$$

$$y-2=\frac{1}{4}(x-6) \rightarrow \boxed{y=\frac{1}{4}x+\frac{1}{2}}$$

(c) $y = \frac{1}{x^2}$ at $(1/2, 4)$

$$\lim_{x\to 1/2} \frac{\frac{1}{x^2}-4}{x-1/2} \cdot \frac{2x^2}{2x^2} = \lim_{x\to 1/2} \frac{2(1-4x^2)}{x^2(2x-1)} = \lim_{x\to 1/2} \frac{2\overset{(-1)}{\cancel{(1-2x)}}(1+2x)}{x^2\cancel{(2x-1)}}$$

$$= \lim_{x\to 1/2} \frac{-2(1+2x)}{x^2} = \frac{-2(2)}{1/4} = \boxed{-16}$$

$$y-4=-16(x-1/2) \rightarrow \boxed{y=-16x+12}$$

8. Use the limit definition of the derivative to find the derivatives of the following functions:

(a) $f(x) = 3x^2 - 2x + 1$

$$\lim_{h \to 0} \frac{(3(x+h)^2 - 2(x+h) + 1) - (3x^2 - 2x + 1)}{h} = \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{- 2x} - 2h \cancel{+1} \cancel{- 3x^2} \cancel{+ 2x} \cancel{- 1}}{h}$$

$$= \lim_{h \to 0} (6x + 3h - 2)$$

$$= \boxed{6x - 2}$$

(b) $g(x) = 3x^5$

$$\lim_{h \to 0} \frac{3(x+h)^5 - 3x^5}{h} = \lim_{h \to 0} \frac{3(\cancel{x^5} + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5) \cancel{- 3x^5}}{h}$$

$$= \lim_{h \to 0} (15x^4 + 30x^3h + 30x^2h^2 + 15xh^3 + 3h^4)$$

$$= \boxed{15x^4}$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$\boxed{1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1}$

9. Use the limit definition of the derivative to find the derivatives of the following functions:

(a) $h(x) = \frac{x+1}{x-1}$

$$\lim_{r \to x} \frac{\frac{r+1}{r-1} - \frac{x+1}{x-1}}{r-x} \cdot \frac{(r-1)(x-1)}{(r-1)(x-1)} = \lim_{r \to x} \frac{(r+1)(x-1) - (x+1)(r-1)}{(r-x)(r-1)(x-1)}$$

$$= \lim_{r \to x} \frac{(\cancel{rx} - r + x \cancel{-1}) - (\cancel{xr} - x + r \cancel{-1})}{(r-x)(r-1)(x-1)}$$

$$= \lim_{r \to x} \frac{-2\cancel{(r-x)}}{\cancel{(r-x)}(r-1)(x-1)}$$

$$= \boxed{\frac{-2}{(x-1)^2}}$$

(b) $k(x) = \frac{2}{\sqrt{x}}$

$$\lim_{r \to x} \frac{\frac{2}{\sqrt{r}} - \frac{2}{\sqrt{x}}}{r-x} \cdot \frac{\sqrt{r}\sqrt{x}}{\sqrt{r}\sqrt{x}} = \lim_{r \to x} \frac{2(\sqrt{x}-\sqrt{r})}{(r-x)\sqrt{r}\sqrt{x}} \cdot \frac{\sqrt{x}+\sqrt{r}}{\sqrt{x}+\sqrt{r}}$$

$$= \lim_{r \to x} \frac{2\cancel{(x-r)}^{(-1)}}{\cancel{(r-x)}\sqrt{r}\sqrt{x}(\sqrt{x}+\sqrt{r})}$$

$$= \frac{-2}{\sqrt{x}\sqrt{x}(\sqrt{x}+\sqrt{x})} = \frac{-2}{2x\sqrt{x}} = \boxed{\frac{-1}{x^{3/2}}}$$

10. Take the derivative of the following functions. (You do not have to use the limit definition.)

(a) $f(x) = 5x^8 - 3x^4 + 2x - 1$

$f'(x) = \boxed{40x^7 - 12x^3 + 2}$

(b) $g(x) = \sqrt{x} - \frac{1}{x^3}$ $= x^{1/2} - x^{-3}$

$g'(x) = \boxed{\frac{1}{2}x^{-1/2} + 3x^{-4}}$

(c) $h(x) = x\sqrt{x}/\sqrt[3]{x}$ $= x \cdot x^{1/2} \cdot x^{-1/3} = x^{7/6}$

$h'(x) = \boxed{\frac{7}{6}x^{1/6}}$