Math 127 Exam 1 Fall 2009

Put your answers on the answer card.

1. Suppose $f(x) = 2x^2 + x + 1$. When you compute

$$\frac{f(x+h) - f(x)}{h}$$

and simplify what do you obtain?

(a) $2x + 1$
(b) $4x + 1 + h$
(c) $2x^2 + 4hx + 2h^2 - x - 1$
(d) $2x^2 + 4x - 2h^2 - 4h$
(e) $2x^2 + 4x - 2h - 4$
(f) $(2x^2 + 4x)/h - 2h - 4$
(g) $(4x + 1)/h$
(h) $(2x^2 + 3xh + 2h^2 + h + 1)/h$
(i) 2
(j) 0

2. Find the equation of the tangent line to $y = \sqrt{x}$ at the point $(4, 2)$

(a) $y - 4x = 4$
(b) $-4y + x = 6$
(c) $-y - 4x = 0$
(d) $4y + x = -4$
(e) $4y - x = -4$
(f) $4y + x = 0$
(g) $4y + x = 4$
(h) $4y - x = 0$
(i) $4y - x = 4$
(j) $4y - x = 6$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

Tan line $y - 2 = m(x - 4)$
$\uparrow$
deriv at $x = 4$

$y - 2 = \frac{1}{4}(x - 4)$
$4y - x = 4$

3. What is the slope of the tangent line to the curve

$$y = \frac{x}{1+x^2}$$

at the point $(0,0)$?

(a) -4
(b) -2
(c) -1
(d) $-1/2$
(e) 0
(f) $1/2$
$\boxed{\text{(g) } 1}$
(h) 2
(i) 4
(j) 8

slope = deriv

$$\frac{dy}{dx} = \frac{(1+x^2) - x(2x)}{(1+x^2)^2}$$

at $x = 0$, $= 1$

4. Find the slope of the tangent line to the curve $y = (\log x)^2$ at $x = 1$.

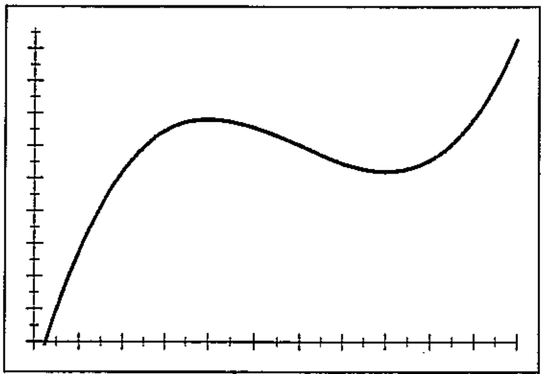
(a) -2
(b) $-e$
(c) -1
$\boxed{\text{(d) } 0}$
(e) 1
(f) e
(g) 2
(h) $1/e$
(i) $-1/e$
(j) $\log 2$

$$y' = 2 \log x \frac{d}{dx} \log x$$

$$= \frac{2 \log x}{x}$$

at $x = 1$ $\quad \frac{2 \cdot 0}{1} = 0$

5. The function $f(x) = x^3 + 3x^2 - 4$ is increasing on the interval $-\infty$ to a, decreasing on the interval from a to b and increasing on the interval from b to ∞.



$y = x^3 + 3x^2 - 4$

The values of a and b are

(a) $-2, 2$
(b) $-1, 2$
(c) $0, 2$
(d) $1, 2$
(e) $0, 1$
(f) $-1, 1$
(g) $-2, 1$
(h) $-2, 0$
(i) $-1, 0$
(j) $-2, -1$

$$y' = 3x^2 + 6x$$
$$= 3(x^2 + 2x)$$
$$= \underbrace{3x(x+2)}$$

zero when $x = 0$ & $x = -2$

for $x < -2$ neg · neg = pos

$-2 < x < 0$ neg · pos = neg

$x > 0$ pos · pos = pos

f' pos $f \uparrow$
f' neg $f \downarrow$
changes at $0, -2$

6.

For $f(x) = x^2 + \frac{1}{x^2}$, find the value at 1 of the third derivative, $f'''(1) = \frac{d^3 f}{dx^3}(1)$.

(a) 24
(b) 16
(c) 12
(d) 8
(e) 4
(f) 0
(g) −4
(h) −8
(i) −16
(j) −24

$f = x^2 + x^{-2}$

$f' = 2x - 2x^{-3}$

$f'' = 2 + 6x^{-4}$

$f''' = 0 - 24x^{-5}$

$x = 1 \longrightarrow -24$

7.

Find $f'(0)$ for the function $f(x) = \left(\sqrt{2x^2+1} + 3x + 7\right)^{1/3}$.

(a) 1/8
(b) 1/4
(c) 1/2
(d) 2/3
(e) 1
(f) 3/2
(g) 2
(h) 3
(i) 4
(j) 12

$f' = \frac{1}{3}(\quad)^{-2/3} \frac{d}{dx}\left(\sqrt{2x^2+1} + 3x + 7\right)$

$= \frac{1}{3}(\quad)^{-2/3}\left[\frac{1}{2\sqrt{2x^2+1}} \cdot 4x + 3\right]$

at $x = 0$

$\frac{1}{3}(1+7)^{-2/3}\left[\frac{1}{2} \cdot 4 \cdot 0 + 3\right]$

$\frac{1}{3}(8^{-2/3})(3) = \frac{1}{4}$

8.

Evaluate $\lim_{x\to 3} \frac{x^2-2x+1}{x^2-4}$

(a) 0
(b) 1/5
(c) 2/5
$\boxed{\text{(d) } 4/5}$
(e) 8/5
(f) 1/3
(g) 2/3
(h) 1
(i) 4/3
(j) 5/3

$x \to 3$ top $\to 3^2 - 2\cdot 3 + 1 = 9 - 6 + 1 = 4$

buttom $\to 3^2 - 4 = 5$

ratio $\to 4/5$

9.

Evaluate $\lim_{x\to 2} \frac{\cancel{x^2-2x+1}\ 2(x^2-x-2)}{x^2-4}$

(a) 4
(b) 2
$\boxed{\text{(c) } 3/2}$
(d) 1
(e) 1/2
(f) 0
(g) −1/2
(h) −1
(i) −3/2
(j) −2

$x \to 0$ top $\to 2(4-2-2) = 0$

but $\to 2^2 - 4 = 0$

need to do algebra first

$$\frac{2(x^2-x-2)}{x^2-4} = \frac{2(x^2-x-2)}{(x-2)(x+2)}$$

$$= \frac{2(x-2)(x+1)}{(x-2)(x+2)}$$

$$= \frac{2(x+1)}{x+2} \xrightarrow[x\to 2]{\text{as}} \frac{2\cdot 3}{4} = \frac{3}{2}$$

10. Find the tangent line to the graph of $y = 3e^{2x}$ at the point $(0, 3)$

(a) $y - 3 = 6x$
(b) $y - 3 = 3x$
(c) $y - 3 = x$
(d) $y - 3 = -3x$
(e) $y - 3 = 2x$
(f) $y + 3 = x$
(g) $y + 3 = 2x$
(h) $y + 3 = 6x$
(i) $y + 3 = -2x$
(j) $y + 3 = -3x$

$y - 3 = m(x - 0)$

deriv $y' = 3e^{2x} \cdot 2$

at $x = 0$, 6

$y - 3 = 6x$

11.

Find $\frac{dZ}{dt}$ for $Z = (1 + 2u^2)^2$ and $u(t) = t^3$.

(a) $(1 + 2t^6)\, 6t^5$
(b) $(1 + 2t^6)\, 24t^6$
(c) $(1 + 2t^6)^2\, 24t^6$
(d) $(1 + 2t^6)^2\, 12t^5$
(e) $(1 + 2t^6)\, 24t^5$
(f) $(1 + 2t^6)\, 12t^5$
(g) $(1 + 2t^6)\, 3t^2$
(h) $(1 + 2t^6)\, 6t^5$
(i) $(1 + 12t^5)^5$
(j) $(1 + 2t^6)\, 12t^5$

$$\frac{dz}{dt} = \frac{dz}{du}\frac{du}{dt}$$

$$= \left(2(1+2u^2) \cdot 4u\right) 3t^2$$

$$= 24(1 + 2t^6)\, t^3 \cdot t^2$$

12. Find the derivative of $f(x) = xe^x - e^x$

(a) $xe^x - x$
(b) $xe^x + x$
(c) $xe^x + e^x - x$
(d) $xe^x - xxe^x - x$
(e) $xe^x - x$
(f) $xe^x + e^x - x$
(g) $xe^x - e^x - x$
(h) $xe^x - e^x$
(i) $xe^x - x$
(j) xe^x

$f' = xe^x + e^x - e^x$

13. Find the equation of the tangent line to the graph of the function $f(x) = x \log x - x$ at the point $(1, -1)$.

(a) $y = 1$
(b) $y = -1$
(c) $y = x + 1$
(d) $y = x - 1$
(e) $y = -x + 1$
(f) $y = -x - 1$
(g) $y = x$
(h) $y = -x$
(i) $1 = x$
(j) $y = x \log x + 1$

$f' = x \cdot \frac{1}{x} + \log x - 1$

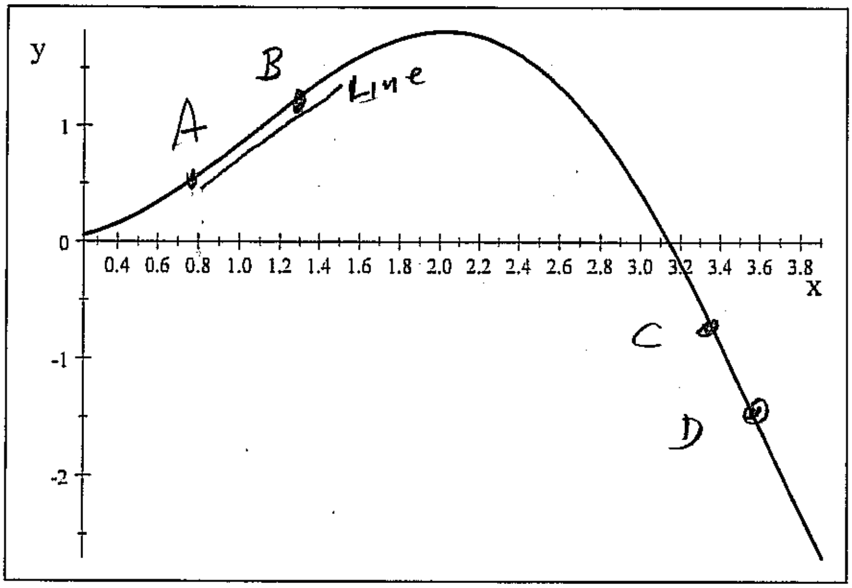
$= \log x$

at $x = 1$ $f' = 0$

$y - (-1) = 0(x - 1)$

$y = -1$

14. The graph of a function $f(x)$ is shown



Which of the following statements appears to be correct?

1	2	3	4	5
$f(.8) > 0$	$\frac{df}{dx}(1.3) > 0$	f is increasing in the interval $(.8, 1.6)$	$f(3.4) > 0$	$\frac{df}{dx}(3.6) > 0$

(a) All
(b) None
(c) 1
(d) 2
(e) 3,4,5
(f) 1,2,5
(g) 1,3
(h) 2,3
(i) 1,2,3
(j) 2,3,4

1 ✓ graph above axis at A
2 ✓ graph ↑ at B
3 ✓ graph ↑ along Line
4 X graph below axis at C
5 X graph ↓ at D

15. Find the slope of the tangent line to the curve defined by $x^2y + xy^2 = 2$ at the point $(1,1)$.

(a) -2
(b) -1
(c) $-1/2$
(d) $-\sqrt{2}$
(e) $-\sqrt{3}$
(f) 0
(g) $\sqrt{5}$
(h) 2
(i) $\sqrt{2}$
(j) 1

Implicit diff

$$x^2 \frac{dy}{dx} + y(2y) + 2y\frac{dy}{dx} \cdot x + y^2 = 0$$

$$x = y = 1$$

$$\frac{dy}{dx} + 2 + 2\frac{dy}{dx} + 1 = 0$$

$$\frac{dy}{dx} = -1$$

16. The function $f(x) = x^7 + 4x^4$ has an inverse function g. Find $g'(5)$.

(a) 11
(b) 13
(c) 1/13
(d) 1/11
(e) -27
(f) -14
(g) $-1/5$
(h) 23
(i) 1/23
(j) -17

$$g'(5) = \frac{1}{f'(1)}$$ because $f(1) = 1^7 + 4 \cdot 1^4 = 5$

so $f(1) = 5$

$g(5) = 1$

$$f'(x) = 7x^6 + 16x^3$$

$$f'(1) = 7 + 16 = 23$$

17. On what interval(s) is the function

$$f(x) = \frac{1}{6 + x^2 + 6x}$$

increasing?

condition for $f \uparrow$

(a) $(-\infty, 3)$
(b) $(-\infty, 0)$
(c) $(-\infty, -3)$
(d) $(-3, 0)$
(e) $(-3, 3)$
(f) $(0, 3)$
(g) $(-3, \infty)$
(h) $(-\infty, -3)$ and $(3, \infty)$
(i) $(-\infty, 0)$ and $(3, \infty)$
(j) $(-\infty, -3)$ and $(0, \infty)$

$$f' = \frac{1}{(6+x^2+6x)^2}(-1)(2x+6)$$

bottom is $(\,)^2$ always positive

so $\boxed{f' \text{ pos}}$ when top pos

so when $-2x - 6 > 0$

$-2x > 6$

$x < -3$

18. Suppose $a(t)$ is a differentiable functions of t and that $a(1) = 1, a(2) = 2$, $a'(1) = 5$, and $a'(2) = 10$. Find the derivative of the function $(a(2t))^2$ at the point $t = 1$.

(a) 0
(b) 1
(c) 2
(d) 4
(e) 5
(f) 10
(g) 20
(h) 40
(i) 80
(j) 120

$$\frac{d}{dt}(a(2t))^2 = 2a(2t)\frac{d}{dt}a(2t)$$

$$= 2a(2t)\,a'(2t)\frac{d}{dt}2t$$

$$= 2a(2t)\,a'(2t)\cdot 2$$

at $t = 1$

$$= 2a(2)a'(2)\cdot 2$$

$$= 4 \cdot 2 \cdot 10$$

19. Using logarithmic differentiation if you wish compute the derivative of the function

$$f(x) = (2x)^x.$$

(a) $(2x)^x$
(b) $2^x x^x$
(c) $(2x)^{x-1}$
(d) $(2x)^{x-1}\,2$
(e) $(2x)^x \log 2x$
$\boxed{\text{(f) } (2x)^x\,(1+\log 2x)}$
(g) $(2x)^x\,(1+2x)$
(h) $(2x)^x\,(1+\log x)$
(i) $2x^x + (2x)^{x-1}$
(j) $x^x 2^{x-1} \log 2$

$\log f(x) = \log((2x)^x)$

$\log f(x) = x \log 2x$

diff each side

$\frac{f'(x)}{f(x)} = x \cdot \frac{1}{2x}2 + \log 2x$

$= 1 + \log 2x$

$f'(x) = f(x)\,(1+\log 2x)$

$= (2x)^x\,(1+\log 2x)$

20.

For $f(t) = \log \frac{(t^2+1)}{\sqrt{t+1}}$ compute the derivative at $t=1$, $f'(1)$.

(a) 1/4
$\boxed{\text{(b) } 3/4}$
(c) 1
(d) 3
(e) 3/2
(f) 2/3
(g) 4/3
(h) 0
(i) 2
(j) 4

$\log\left(\frac{t^2+1}{\sqrt{t+1}}\right) = \log(t^2+1) - \frac{1}{2}\log(t+1)$

$\frac{d}{dt}\left(\quad \right) = \frac{1}{t^2+1} \cdot 2t - \frac{1}{2}\,\frac{1}{t+1}$

at $t=1$ $\quad \frac{1}{2}\cdot 2 - \frac{1}{2}\,\frac{1}{1+1} = 1 - \frac{1}{4} = \frac{3}{4}$