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Discussion Section:

This exam has 12 questions:

- 10 multiple choice worth 6 points each.
- 2 hand graded worth 20 points each.

Important:

- No graphing calculators!
- For the multiple choice questions, mark your answer on the answer card.
- Show all your work for the written problems. You will be graded on the ease of reading your solution.
- You are allowed a  $3 \times 5$  note card for the exam.

1. Find the value of  $x$  at which the function  $f(x) = -x^3 - 4x + 8$  has a relative maximum.

- (a)  $-3$
- (b)  $-2$
- (c)  $-\sqrt{2}$
- (d)  $-1$
- (e)  $0$
- (f)  $1$
- (g)  $\sqrt{2}$
- (h)  $2$
- (i)  $3$
- (j) There is no relative maximum.

2. The graph of  $y = \frac{3x^2+1}{x+1}$  has a slant asymptote. What is the equation of the slant asymptote?
- (a)  $y = 3x - 2$
  - (b)  $y = x + 1$
  - (c)  $y = x + 3$
  - (d)  $y = 3x + 1$
  - (e)  $y = 3x$
  - (f)  $y = 3$
  - (g)  $y = 2x$
  - (h)  $y = 3x - 3$
3. Consider the equation  $y = x(25 - x^2)$ . Which of the following best describes the graph of  $y$  when  $x = 3$ ?
- (a) increasing, concave down
  - (b) increasing, concave up
  - (c) decreasing, concave down
  - (d) decreasing, concave up
  - (e) local maximum
  - (f) local minimum
  - (g) critical point, inflection point
  - (h) increasing, inflection point
  - (i) decreasing, inflection point
  - (j) not differentiable at this point
4. The Canonikon company produces digital cameras. The demand function for their cameras is  $P(x) = 400 - 0.4x$ , where  $P(x)$  is the price (in dollars) they must charge in order to sell  $x$  cameras. The cost of producing  $x$  cameras is  $C(x) = 2000 + 160x$ . At what price should they sell the cameras in order to maximize their profit?
- (a) 160
  - (b) 200
  - (c) 280
  - (d) 320
  - (e) 360
  - (f) 375
  - (g) 400
  - (h) 420
  - (i) 480
  - (j) 600

5. Let  $y = \frac{x}{x^2+1}$ . What is the maximum value of  $y$ ?

- (a)  $-1/2$
- (b)  $-1/4$
- (c)  $0$
- (d)  $2/5$
- (e)  $3/8$
- (f)  $1/4$
- (g)  $1/2$
- (h)  $1$
- (i)  $2$
- (j) There is no absolute maximum value.

6. It costs \$5000 to produce 50 Ionocaster guitars, and the marginal cost at that point is \$75. Estimate the cost of producing 53 guitars.

- (a) \$5000
- (b) \$5025
- (c) \$5075
- (d) \$5100
- (e) \$5125
- (f) \$5150
- (g) \$5200
- (h) \$5225
- (i) \$5300
- (j) \$5350

7. Suppose that  $x$  and  $y$  are related by the equation  $x^2 - 1 = y^3$ . Find the equation of the tangent line at the point  $(3, 2)$ .

- (a)  $y = x/2 + 1/2$
- (b)  $y = 2x/3$
- (c)  $y = 3x/2$
- (d)  $y = 2x - 4$
- (e)  $y = x - 1$
- (f)  $y = 2$
- (g)  $y = x/3 + 1$
- (h)  $y = x/4 + 5/4$
- (i)  $y = -2x + 8$

8. Helium is being pumped into a spherical balloon at the rate of  $4\pi$  cubic inches per second. Currently, the radius of the balloon is 1 inch. How fast (in inches/second) is the radius of the balloon increasing?

- (a)  $1/3$
- (b)  $3/4$
- (c)  $\pi/4$
- (d) 1
- (e)  $4/3$
- (f) 2
- (g)  $3\pi/4$
- (h)  $\pi$
- (i)  $4\pi/3$
- (j)  $4\pi^2$

9. What is the slope of the graph of  $y = x(x^2 + 1)(x^2 - 1)$  when  $x = 1$ ?

- (a)  $-2$
- (b)  $-1$
- (c) 0
- (d) 1
- (e) 2
- (f) 3
- (g) 4
- (h) 5
- (i) 6
- (j) 8

10. Suppose that  $f(x) = x^3 - 6x^2 + 9x$ . Find the absolute maximum and minimum values of  $f(x)$  on the interval  $[0, 5]$ .

- (a) Min:  $-9$ , Max: 4
- (b) Min:  $-9$ , Max: 1
- (c) Min: 0, Max: 4
- (d) Min: 0, Max: 1
- (e) Min:  $-16$ , Max: 4
- (f) Min:  $-16$ , Max: 20
- (g) Min:  $-9$ , Max: 20
- (h) Min:  $-8$ , Max: 1
- (i) Min: 0, Max: 20
- (j) Min:  $-8$ , Max: 4

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11. Consider the curve  $y = (x^2 - 9)^{2/3}$ .

(a) Compute  $dy/dx$ .

(b) There is one value of  $x$  where  $dy/dx = 0$ , and there are two values of  $x$  where  $dy/dx$  is not defined. What are these three critical values?

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- (c) At each of the three values you found in part (b), check if  $y$  has a local maximum, local minimum, or neither.

- (d) Draw a sketch of the graph of  $y$ .

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12. The demand function for a certain special edition book is  $P(x) = \frac{200}{x} - \frac{100}{x^2}$ , where  $P(x)$  is the price (in dollars) at which the publisher can sell  $x$  copies of the book. The cost of printing  $x$  copies is  $C(x) = x + 100$ .

(a) Write down equations for the revenue function  $R(x)$  and the profit function  $\Pi(x)$ .

(b) How many copies should the publisher print in order to maximize profit?

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- (c) What is the maximum profit that the publisher can earn from printing this book?
- (d) Draw a sketch of the profit function  $\Pi(x)$ , being sure to include important features such as local maxima/minima, inflection points, and asymptotes.