

This exam has 20 questions; indicate your answers on the answer card.

1. Use the method of linear approximation with the function $f(x) = \sqrt{x}$ and base point 100 to estimate $\sqrt{99}$. The estimate is
 - a. 9.93333
 - b. 9.94444
 - c. 9.94555
 - d. 9.94666
 - e. 9.94678
 - f. 9.94987
 - g. 9.95
 - h. 9.96
 - i. 9.97
 - j. 9.98

2. Compute the differential $d(8x^4 + 1)$
 - a. $8x^4 dx$
 - b. $8x^4$
 - c. $(8x^4 + 1)dx$
 - d. $(8x^4 + 1)$
 - e. $32x^3$
 - f. $32x^3 dx$
 - g. $(32x^3 + 1)dx$
 - h. $32x^3 + 1$
 - i. $8x^4 + dx$
 - j. 0

3. What is the quadratic (i.e. 2nd degree) approximation to the function $f(x) = x^{2/3}$ at the base point $x = 1$?

a. $1 + \frac{2}{3}(x-1) - \frac{1}{9}(x-1)^2$

b. $1 + \frac{2}{3}(x-1) + \frac{1}{9}(x-1)^2$

c. $1 + \frac{2}{3}(x-1) - \frac{2}{9}(x-1)^2$

d. $1 + \frac{2}{3}(x-1) + \frac{2}{9}(x-1)^2$

e. $1 + \frac{2}{3}(x-1)$

f. $1 - \frac{1}{3}(x-1)$

g. $1 + \frac{1}{9}(x-1)^2$

h. $1 - \frac{1}{9}(x-1)^2$

i. $1 + \frac{2}{9}(x-1)^2$

j. 1

4. What is the numerical value of the estimate of $\exp(-1) = e^{-1}$ that you obtain by using the second degree Taylor polynomial estimate for $f(x) = e^x$ at the base point $x = 0$?

a. 1.11111

b. 1.109090

c. 1.11

d. 1.15

e. 1.105

f. 1.05

g. 0.995

h. .95

i. .95555

j. 1

5. For the function $f(x) = 17x^4$ compute the elasticity $El_x f(x)$.
- a. 0
 - b. 1
 - c. $1/x$
 - d. 4
 - e. $4/x$
 - f. 17
 - g. 68
 - h. $68x^3$
 - i. $17x^4$
 - j. It has no elasticity.
6. Suppose $D(P)$ is the demand function for a product as a function of price. Suppose that the price elasticity of demand, $El_P D(P)$ is -0.5 . Approximately what percentage change in demand would you expect if the price were to increase by 2%?
- a. Increase by 4%
 - b. Increase by 2%
 - c. Increase by 1%
 - d. Increase by .5%
 - e. Decrease by 4%
 - f. Decrease by 2%
 - g. Decrease by 1%
 - h. Decrease by .5%
 - i. No change.
 - j. Can't make a reasonable estimate from the given information.

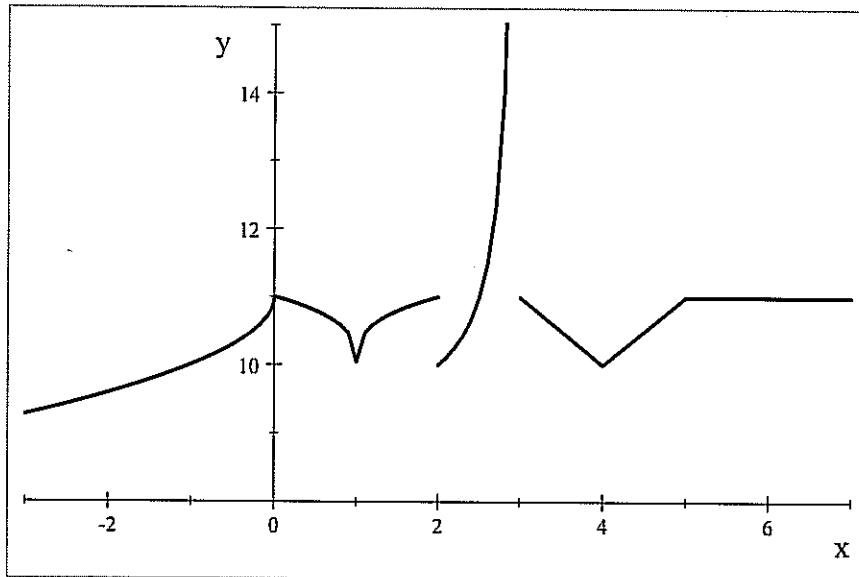
7. For what values of x is the function

$$\frac{x^2 + 1}{2 - \sqrt{x}}$$

continuous?

- a. All x
- b. All x except $x = 4$
- c. All x except $x = 4$ and $x = -4$
- d. All $x \geq 0$
- e. All $x \geq 0$ except for $x = 2$
- f. All $x \geq 0$ except for $x = 4$
- g. All $x > 2$
- h. All $x > 4$
- i. All $x < 4$
- j. No x

8. The graph of the function $f(x)$ is shown. For which of the values $x = -1, 0, 1, 2, 3, 4, 5, 6$ does it appear that the function is NOT continuous?



$$y = f(x)$$

- a. All of them.
- b. None of them
- c. $x = 0, 1$
- d. $x = 2, 3$
- e. $x = 4, 5$
- f. $x = -1, 6$
- g. $x = 3, 5$
- h. $x = 1$
- i. $x = 2$
- j. $x = 3$

9. Suppose you use Newton's method to estimate $\sqrt{3}$ by estimating a solution of the equation $f(x) = x^2 - 3$ using the starting estimate $x_0 = 1$. What is your second revised estimate, x_2 ?
- a. 0
 - b. 1
 - c. 2
 - d. $\sqrt{3}$
 - e. 1.732
 - f. 1.5
 - g. 1.66666
 - h. 1.75
 - i. 1.875
 - j. 1.888

10. Evaluate

$$\lim_{n \rightarrow \infty} \left(3n^2 - \frac{1}{n^3} \right).$$

- a. 0
- b. 1
- c. 2
- d. 3
- e. $1/3$
- f. $1/2$
- g. $-1/3$
- h. $-1/2$
- i. Cannot be determined.
- j. ∞ , that is, there is no ordinary limit but the expression becomes and remains larger than any bound.

11. Evaluate

$$\lim_{x \rightarrow 0} \frac{(1-x)^{2/3} - 1}{x}.$$

- a. 0
- b. 1
- c. 2
- d. 3
- e. 1/3
- f. 2/3
- g. -1/3
- h. -2/3
- i. Cannot be determined.
- j. ∞ , that is, there is no ordinary limit but the expression becomes and remains larger than any bound.

12. Evaluate

$$\lim_{x \rightarrow 0} \frac{e^{-2x} - 1 + 2x}{3x^2}.$$

- a. 0
- b. 1
- c. 2
- d. 3
- e. 2/3
- f. 1/2
- g. -1/3
- h. -2/3
- i. Cannot be determined.
- j. ∞ , that is, there is no ordinary limit but the expression becomes and remains larger than any bound.

13. Find the maximum and minimum values of the function

$$f(x) = \frac{1}{2+x^2}.$$

- a. The maximum value is 0, the minimum value is $1/2$.
 - b. The maximum value is 1, the minimum value is $1/2$.
 - c. The maximum value is $1/2$, the minimum value is 0.
 - d. There is no maximum value, there is no minimum value.
 - e. The maximum value is $1/2$, there is no minimum value.
 - f. There is no maximum value, the minimum value is -1 .
 - g. There is no maximum value, the minimum value is 0.
 - h. The maximum value is 2, there is no minimum.
 - i. The maximum value is 1, there is no minimum value.
 - j. The maximum value is 1, the minimum value is $1/2$.
14. Find the maximum and minimum values of the function $f(x) = \sqrt{x-3} + 100$, $x \geq 3$.
- a. There is no maximum, there is no minimum.
 - b. There is no maximum, the minimum is 0.
 - c. The maximum is 100, there is no minimum.
 - d. The maximum is 0, the minimum is -3 .
 - e. There is no maximum, the minimum is 100.
 - f. The maximum is 100, the minimum is 0.
 - g. The maximum is $\sqrt{97}$, the minimum is 0.
 - h. The maximum is 100, the minimum is $\sqrt{97}$.
 - i. The maximum is 100, the minimum is 100.
 - j. There is no maximum, the minimum is $\sqrt{97}$.

15. Find the maximum value of the function

$$c(t) = \frac{t}{t^2 + 4}, \quad t \geq 0.$$

- a. 0
 - b. 1
 - c. 2
 - d. 3
 - e. 4
 - f. 1/2
 - g. 1/3
 - h. 1/4
 - i. 3/4
 - j. There is no maximum
16. The cost of producing Q units of a commodity is

$$C(Q) = Q^2 + 5Q + 16.$$

Find the value of Q which minimizes $A(Q) = C(Q)/Q$, the average cost per unit.

- a. 0
- b. 4
- c. 3
- d. 2
- e. 1
- f. ± 4
- g. ± 2
- h. 3/2
- i. 2/3
- j. There is no such value.

17. Find the maximum and minimum values of the convex function

$$f(x) = x + e^{-x}.$$

- a. No maximum, no minimum.
 - b. No maximum, minimum value 0.
 - c. No maximum, minimum value 1.
 - d. No maximum, minimum value e .
 - e. Maximum value 0, no minimum.
 - f. Maximum value 1, no minimum.
 - g. Maximum value $1 + 1/e$, no minimum.
 - h. Maximum value $1 + 1/e$, minimum value 0.
 - i. Maximum value e , minimum value 1.
 - j. Maximum value e , minimum value 0.
18. A firm produces $Q = 2\sqrt{L}$ units of a commodity when L units of labor are employed. If the price obtained per unit is \$160 and the price per unit of labor is \$40, what value of L maximizes profits?
- a. 2
 - b. 4
 - c. 6
 - d. 8
 - e. 10
 - f. 12
 - g. 14
 - h. 16
 - i. 18
 - j. 20

19. What conclusion can you draw about a function $f(x)$ if you know it has the following three properties?

$$f'(x) < 0 \text{ for all } x < 3$$

$$f'(3) = 0$$

$$f'(x) > 0 \text{ for all } x > 3$$

- a. The function has no maximum and no minimum.
 - b. The function has a maximum at $x = 3$ and no minimum
 - c. The function has a minimum at $x = 3$ and no maximum.
 - d. The function has a maximum at $x = 3$, there is not enough information to tell if it has a minimum.
 - e. The function has a minimum at $x = 3$, there is not enough information to tell if the function has a maximum.
 - f. There is not enough information to know if the function has a maximum or if it has a minimum.
20. Evaluate

$$\lim_{x \rightarrow 2} \frac{x^2 + 2x - 1}{x^2 - 2x + 1}$$

- a. 1
- b. 3
- c. 5
- d. 7
- e. 9
- f. 0
- g. -1
- h. There is no limit.
- i. Cannot be determined.
- j. ∞ , that is, there is no ordinary limit but the expression becomes and remains larger than any bound