Math 127 Exam 2 Fall '09

This exam has 20 question; indicate your answers on the answer card.

1. Use the method of linear approximation with the function $f(x) = \sqrt{x}$ and base point 100 to estimate $\sqrt{99}$. The estimate is

a. 9.93333

b. 9.94444

c. 9.94555

d. 9.94666

e. 9.94678

f. 9.94987

g. 9.95

h. 9.96

i. 9.97

j. 9.98

$f(x) \sim f(a) + f'(a)(x-a)$

$\sqrt{99} \sim \sqrt{100} + \frac{1}{2\sqrt{100}}(99-100)$

$= 10 + \frac{1}{20}(-1) = 10 - .05$

$= 9.95$

2. Compute the differential $d(8x^4 + 1)$

a. $8x^4dx$

b. $8x^4$

c. $(8x^4 + 1)dx$

d. $(8x^4 + 1)$

e. $32x^3$

f. $32x^3dx$

g. $(32x^3 + 1)dx$

h. $32x^3 + 1$

i. $8x^4 + dx$

j. 0

$d f(x) = f'(x) dx$

$d(8x^4+1) = 32x^3 dx$

3. What is the quadratic (i.e. 2^{nd} degree) approximation to the function $f(x) = x^{2/3}$ at the base point $x = 1$?

a. $\boxed{1+\frac{2}{3}(x-1)-\frac{1}{9}(x-1)^2}$

b. $1+\frac{2}{3}(x-1)+\frac{1}{9}(x-1)^2$

c. $1+\frac{2}{3}(x-1)-\frac{2}{9}(x-1)^2$

d. $1+\frac{2}{3}(x-1)+\frac{2}{9}(x-1)^2$

e. $1+\frac{2}{3}(x-1)$

f. $1-\frac{1}{3}(x-1)$

g. $1+\frac{1}{9}(x-1)^2$

h. $1-\frac{1}{9}(x-1)^2$

i. $1+\frac{2}{9}(x-1)^2$

j. 1

$$f(x) \sim f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2$$

$$x^{2/3} \sim 1^{2/3} + \frac{2}{3}1^{-1/3}(x-1) + \frac{1}{2}\cdot\frac{2}{3}\left(\frac{-1}{3}\right)1^{-4/3}(x-1)^2$$

$$= 1 + \frac{2}{3}(x-1) - \frac{1}{9}(x-1)^2$$

4. What is the numerical value of the estimate of $\exp(.1) = e^{.1}$ that you obtain by using the second degree Taylor polynomial estimate for $f(x) = e^x$ at the base point $x = 0$?

a. 1.11111

b. 1.109090

c. 1.11

d. 1.15

e. $\boxed{1.105}$

f. 1.05

g. 0.995

h. .95

i. .95555

j. 1

$$e^x \sim 1 + x + \frac{x^2}{2}$$

$$e^{.1} \sim 1 + .1 + \frac{.01}{2}$$

$$= 1.1 + .005$$

5. For the function $f(x) = 17x^4$ compute the elasticity $\mathrm{El}_x f(x)$.

a. 0

b. 1

c. $1/x$

d. 4

e. $4/x$

f. 17

g. 68

h. $68x^3$

i. $17x^4$

j. It has no elasticity.

$$\mathrm{El}_x f(x) = \frac{x}{f(x)} f'(x)$$

$$= \frac{x}{17x^4} \cdot 17 \cdot 4x^3$$

$$= 4$$

6. Suppose $D(P)$ is the demand function for a product as a function of price. Suppose that the price elasticity of demand, $\mathrm{El}_P D(P)$ is $-.5$ Approximately what percentage change in demand would you expect if the price were to increase by 2%?

a. Increase by 4%

b. Increase by 2%

c. Increase by 1%

d. Increase by .5%

e. Decrease by 4%

f. Decrease by 2%

g. Decrease by 1%

h. Decrease by .5%

i. No change.

j. Can't make a reasonable estimate from the given information.

% chg demand $\approx$ (% chg price) $\times$ El.

$= (2\%)(-\frac{1}{2})$

7. For what values of x is the function

$$\frac{x^2+1}{2-\sqrt{x}}$$

continuous?

a. All x

b. All x except $x = 4$

c. All x except $x = 4$ and $x = -4$

d. All $x \geq 0$

e. $\boxed{\text{All } x \geq 0 \text{ except for } x = 2}$

f. All $x \geq 0$ except for $x = 4$

g. All $x > 2$

h. All $x > 4$

i. All $x < 4$

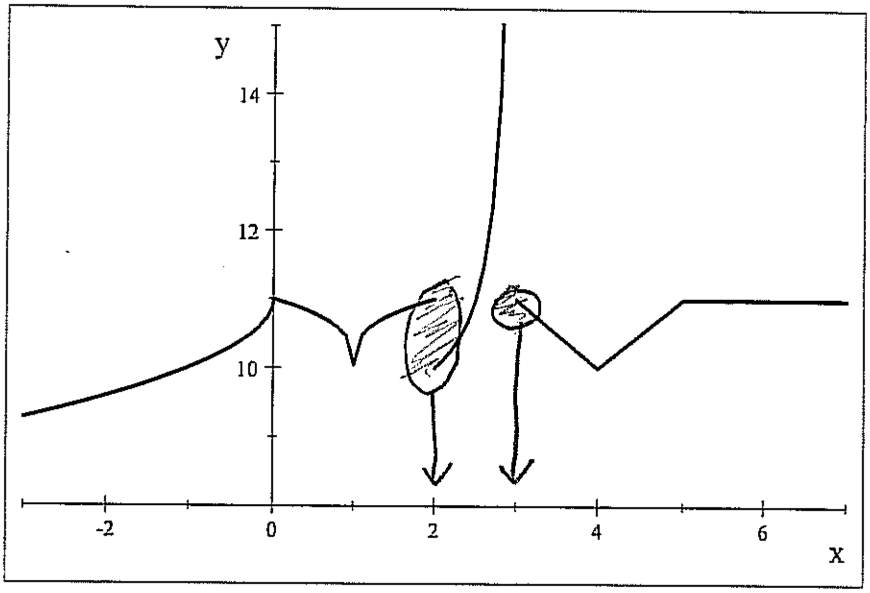
j. No x

This function is continuous wherever formula is properly defined

so $x \geq 0$ for $\sqrt{x}$

$\sqrt{x} \neq 2$ for division

8. The graph of the function $f(x)$ is shown. For which of the values $x = -1, 0, 1, 2, 3, 4, 5, 6$ does it appear that the function is NOT continuous?



$y = f(x)$

a. All of them.

b. None of them

c. $x = 0, 1$

d. $x = 2, 3$

e. $x = 4, 5$

f. $x = -1, 6$

g. $x = 3, 5$

h. $x = 1$

i. $x = 2$

j. $x = 3$

only two places graph is not a curve

9. Suppose you use Newton's method to estimate $\sqrt{3}$ by estimating a solution of the equation $f(x) = x^2 - 3$ using the starting estimate $x_0 = 1$. What is your second revised estimate, x_2?

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^2 - 3}{2x_n}$$

$x_0 = 1$ $\quad x_1 = 1 - \frac{1-3}{2} = 2$

$x_2 = \quad x_2 = 2 - \frac{4-3}{4} = 2 - \frac{1}{4}$

a. 0

b. 1

c. 2

d. $\sqrt{3}$

e. 1.732

f. 1.5

g. 1.66666

h. 1.75

i. 1.875

j. 1.888

10. Evaluate

$$\lim_{n\to\infty}\left(3n^2 - \frac{1}{n^3}\right).$$

n large

$3n^2$ very large positive

$\frac{1}{n^3} \sim 0$

a. 0

b. 1

c. 2

d. 3

e. 1/3

f. 1/2

g. −1/3

h. −1/2

i. Cannot be determined.

j. ∞, that is, there is no ordinary limit but the expression becomes and remains larger than any bound.

11. Evaluate

$$\lim_{x\to 0} \frac{(1-x)^{2/3}-1}{x}.$$

a. 0

b. 1 setting $x = 0$ gives $\frac{0}{0}$

c. 2

d. 3 use l'H

e. 1/3

f. $\boxed{2/3}$ $\lim = \lim_{x\to 0} \frac{2/3\,(1-x)^{-1/3}}{1} = \frac{2}{3}$

g. −1/3

h. −2/3

i. Cannot be determined.

j. ∞, that is, there is no ordinary limit but the expression becomes and remains larger than any bound.

12. Evaluate

$$\lim_{x\to 0} \frac{e^{-2x}-1+2x}{3x^2}.$$

a. 0 again setting $x = 0$ gives 0/0

b. 1

c. 2 after l'H

d. 3 $\text{Ans} = \lim \frac{-2e^{-2x}+2}{6x}$

e. $\boxed{2/3}$ need l'H second time

f. 1/2

g. −1/3 $\text{Ans} = \lim \frac{4e^{-2x}}{6} = \frac{4}{6}$

h. −2/3

i. Cannot be determined.

j. ∞, that is, there is no ordinary limit but the expression becomes and remains larger than any bound.

13. Find the maximum and minimum values of the function

$$f(x) = \frac{1}{2+x^2}.$$

a. The maximum value is 0, the minimum value is 1/2.

b. The maximum value is 1, the minimum value is 1/2.

c. The maximum value is 1/2, the minimum value is 0.

d. There is no maximum value, there is no minimum value.

e. The maximum value is 1/2, there is no minimum value.

f. There is no maximum value, the minimum value is –1.

g. There is no maximum value, the minimum value is 0.

h. The maximum value is 2, there is no minimum.

i. The maximum value is 1, there is no minimum value.

j. The maximum value is 1, the minimum value is 1/2.

bottom is always ≥ 2
so fraction is $\leq \frac{1}{2}$
fraction $= 1/2$ when $x = 0$
so $\boxed{\text{Max} = 1/2}$
bottom gets arbitrarily large so fraction arbitrarily close to zero, but $\frac{1}{\text{something}}$ is never $= 0$
$\boxed{\text{no min}}$

14. Find the maximum and minimum values of the function $f(x) = \sqrt{x-3} + 100$, $x \geq 3$.

a. There is no maximum, there is no minimum.

b. There is no maximum, the minimum is 0.

c. The maximum is 100, there is no minimum.

d. The maximum is 0, the minimum is -3.

e. There is no maximum, the minimum is 100.

f. The maximum is 100, the minimum is 0.

g. The maximum is $\sqrt{97}$, the minimum is 0.

h. The maximum is 100, the minimum is $\sqrt{97}$.

i. The maximum is 100, the minimum is 100.

j. Theere is no maximum, the minimum is $\sqrt{97}$.

$f(x) = 100 + \text{something} \geq 0$
and $f(3) = 100$
so $\boxed{100 = \text{min}}$

max?
$f(x) = 100 + \sqrt{x-3}$
$+ \sqrt{x-3}$ gets arbitrarily large
$\boxed{\text{no max}}$

15. Find the maximum value of the function

$$c(t) = \frac{t}{t^2+4}, \quad t \geq 0.$$

a. 0

b. 1

c. 2

d. 3

e. 4

f. 1/2

g. 1/3

$\boxed{\textbf{h.}\ 1/4}$

i. 3/4

j. There is no maximum

$$c'(t) = \frac{(t^2+4)1 - t(2t)}{(t^2+4)^2}$$

$$= \frac{4-t^2}{(t^2+4)^2}$$

only interested in $t \geq 0$, In that range

$$c'(t) = \frac{\left(\begin{matrix}\text{pos } t \leq 2 \\ \text{neg } t > 2\end{matrix}\right)}{\text{pos}}$$

so graph

and max is at $t = 2$

$c(2) = \frac{2}{8}$

16. The cost of producing Q units of a commodity is

$$C(Q) = Q^2 + 5Q + 16.$$

Find the value of Q which minimizes $A(Q) = C(Q)/Q$, the average cost per unit.

a. 0

$\boxed{\textbf{b.}\ 4}$

c. 3

d. 2

e. 1

f. ± 4

g. ± 2

h. 3/2

i. 2/3

j. There is no such value.

$$A(Q) = \frac{C(Q)}{Q} = \frac{Q^2+5Q+16}{Q}$$

$$= Q + 5 + \frac{16}{Q}$$

$$A'(Q) = 1 - \frac{16}{Q^2}$$

$A' = 0$ for $Q = \pm 4$

but $Q = -4$ not allowed — we are interested in $Q \geq 0$

so $Q = 4$ is candidate

check $A' < 0 \quad 0 < Q < 4$

$A' > 0 \quad Q > 4$

so $Q = 4$ gives min

"Find the value of Q ..." } Ans = 4

17. Find the maximum and minimum values of the convex function

$$f(x) = x + e^{-x}.$$

- **a.** No maximum, no minimum.
- **b.** No maximum, minimum value 0.
- $\boxed{\textbf{c.}}$ No maximum, minimum value 1.
- **d.** No maximum, minimum value e.
- **e.** Maximum value 0, no minimum.
- **f.** Maximum value 1, no minimum.
- **g.** Maximum value $1 + 1/e$, no minimum.
- **h.** Maximum value $1 + 1/e$, minimum value 0.
- **i.** Maximum value e, minimum value 1.
- **j.** Maximum value e, minimum value 0.

because it is convex it will have no max and will have min at stat. pt, if there is one

$f' = 1 - e^{-x}$

$f' = 0$ when $e^{-x} = 1$

when $x = 0$

$f(0) = 0 + e^{-0} = \underline{1}$ is min

18. A firm produces $Q = 2\sqrt{L}$ units of a commodity when L units of labor are employed. If the price obtained per unit is \$160 and the price per unit of labor is \$40, what value of L maximizes profits?

- **a.** 2
- **b.** 4
- **c.** 6
- **d.** 8
- **e.** 10
- **f.** 12
- **g.** 14
- $\boxed{\textbf{h.} \ 16}$
- **i.** 18
- **j.** 20

$\pi(L) = \text{price} \cdot \text{quantity} - (\text{labor rate})\binom{\text{Amt}}{\text{labor}}$

$= 160 \cdot 2\sqrt{L} - 40L$

$\pi' = \dfrac{320}{2\sqrt{L}} - 40$

$\pi'' = \dfrac{320}{2}\left(-\dfrac{1}{2}\right)\dfrac{1}{L^{3/2}} < 0$ when $L \geq 0$

so concave.

so one stat. pt. gives max

stat pt when $\dfrac{320}{2\sqrt{L}} - 40 = 0$

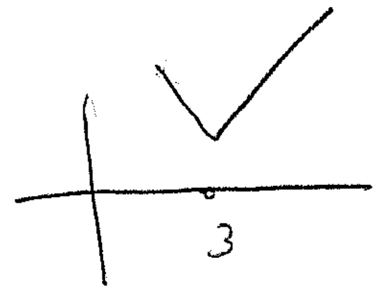
$\sqrt{L} = 4$

$\underline{L = 16}$

540(c)

19. What conclusion can you draw about a function $f(x)$ if you know it has the following three properties?

$$f'(x) < 0 \text{ for all } x < 3$$
$$f'(3) = 0$$
$$f'(x) > 0 \text{ for all } x > 3$$



a. The function has no maximum and no minimum.

b. The function has a maximum at $x = 3$ and no minimum

c. The function has a minimum at $x = 3$ and no maximum.

d. The function has a maximum at $x = 3$, there is not enough information to tell if it has a minimum.

e. The function has a minimum at $x = 3$, there is not enough information to tell if the function has a maximum.

f. There is not enough information to know if the function has a maximum or if it has a minimum.

20. Evaluate

$$\lim_{x \to 2} \frac{x^2 + 2x - 1}{x^2 - 2x + 1}$$

a. 1

b. 3

trying direct substitution gives $\frac{4+4-1}{4-4+1} = 7$

c. 5

d. 7

e. 9

l'H not appropriate & will give wrong answer

f. 0

g. –1

h. There is no limit.

i. Cannot be determined.

j. ∞, that is, there is no ordinary limit but the expression becomes and remains larger than any bound