

Name: \_\_\_\_\_

**Instructions:** You have 2 hours to take this exam. You may use one  $3 \times 5$  notecard. Calculators are **not** allowed.

Show your work and simplify your answers.

1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
7	/10
8	/10
9	/10
10	/10
Total	/100

1. (a) Find the maximum and minimum values of the function  $f(x, y) = x^2 + 3y^2$  subject to the constraints  $x + y = 1$ ,  $x \geq 0$ ,  $y \geq 0$ .

(b) Find the maximum area of a rectangle that can be inscribed between the curve  $y = 1 - x^2$  and the  $x$ -axis, between  $x = -1$  and  $x = 1$ .

2. The volume of a cylinder of height  $h$  and base radius  $r$  is  $\pi r^2 h$ , and its surface area is  $2\pi r^2 + 2\pi r h$ . What are the dimensions of a cylinder of the largest possible volume with surface area  $10\pi \text{ cm}^2$ ?

3. Suppose the demand function for some product is determined to be  $p(x) = 40 - .02x$ , and the cost function is  $C(x) = 8000 + 4x$ .

(a) At what price should the product be sold to maximize revenue, and what would the revenue be in this case?

(b) At what price should the product be sold to maximize profit, and how many units would be sold in this case?

4. Simplify, if possible.

(a)  $\ln\left(\frac{1}{\sqrt{e}}\right)$

(b)  $\ln(1)$

(c)  $\ln(x - y)$

(d)  $\ln(x^3) - \ln(x^2)$

(e)  $\ln x - \ln(1/x)$

5. Solve for  $x$ , if possible.

(a)  $2e^{3x} = 4$

(b)  $5^x = 50$

(c)  $\ln(3x + 1) = 2$

(d)  $\ln(x + 1) - \ln x = 1$

(e)  $3e^{7x} = 5e^{5x}$

6. A bacteria colony grows exponentially. At 12:00 there are 4,000 bacteria and at 3:00 there are 6,000 bacteria.

(a) Write a formula for the number of bacteria  $t$  hours after 12:00.

(b) At what time will there be 9,000 bacteria?

7. (a) \$1,000 is invested in an account that pays 4% interest. How long must we wait to get \$1,500, if the interest is compounded (a) yearly; (b) monthly; (c) continuously?

(b) How much money must be invested now to get \$1,500 in 10 years, if the account yields 4% interest compounded (a) yearly; (b) monthly; (c) continuously?

8. Find the derivatives of the following functions.

(a)  $3e^{4x}$

(b)  $3e^x$

(c)  $xe^{x^2}$

(d)  $e^{x \sin x}$

(e)  $\frac{e^x + e^{-x}}{2}$

9. Find the derivative of the function

$$f(x) = e^{x^5 \tan(3x^2-1)} \sin(e^{5x}).$$

10. (a) Find the intervals on which the function  $y = e^{-x^2}$  is increasing and decreasing.

(b) Find the intervals on which the function  $y = e^{-x^2}$  is concave up and concave down.