Name: 
ID: 
Discussion Section: 
This exam has 20 multiple choice questions: 
Important:

- No graphing calculators!
- Mark your answer on the answer card.
- You are allowed a 4 × 6 note card for the exam.

1. Let \( f(x) = \sqrt{x} \) and \( g(x) = 1 - x^2 \). What is the domain of \((f \circ g)(x)\)?

   (a) \((-\infty, \infty)\)
   (b) \((-\infty, -1] \cup [1, \infty)\)
   (c) \((-\infty, -1) \cup (1, \infty)\)
   (d) \([-1, 1]\)
   (e) \((-1, 1)\)
   (f) \([-1, \infty)\)
   (g) \([0, \infty)\)
   (h) \((0, \infty)\)
   (i) \([1, \infty)\)

   (d) \(f \circ g(x) = \sqrt{1 - x^2}\), which is only defined when \(x\) is between \(-1\) and 1 inclusive.

2. Which of the following best describes the function

\[
f(x) = \begin{cases} 
  x + 1 & \text{if } x < 0; \\
  e^x & \text{if } x \geq 0
\end{cases}
\]

near \(x = 0\)?

   (a) Doesn’t have a limit as \(x \to 0\)
   (b) Has a limit as \(x \to 0\), but not continuous
   (c) Continuous, but not differentiable
   (d) Differentiable, but not continuous
   (e) Continuous and differentiable
   (f) Asymptote

   (e) The function is continuous at \(x = 0\), since \(x + 1\) and \(e^x\) both approach 1 as \(x \to 0\). The derivative of \(x + 1\) is 1, and the derivative of \(e^x\) is \(e^x\). The function is differentiable at \(x = 0\), since 1 and \(e^x\) both approach 1 as \(x \to 0\).
3. Find the limit:
\[
\lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1}
\]
(a) $-1$
(b) $-1/2$
(c) $-1/3$
(d) 0
(e) $1/3$
(f) $1/2$
(g) $3/4$
(h) 1
(i) DNE

(f)
\[
\frac{\sqrt{x} - 1}{x - 1} \cdot \frac{\sqrt{x} + 1}{\sqrt{x} + 1} = \frac{x - 1}{(x - 1)(\sqrt{x} + 1)} = \frac{1}{\sqrt{x} + 1},
\]
which approaches $1/2$ as $x \to 1$.

4. A gardener wants to fence in a rectangular area and then divide the area in half with a fence down the middle parallel to one side (see the diagram below). If the gardener has 60m of fencing material, what is the largest possible area (in m$^2$) of the total fenced-in region?

(a) 60
(b) 120
(c) 150
(d) 180
(e) 225
(f) 240
(g) 250
(h) 300
(i) 500
(j) 900
(c) Let $x$ be the width and $y$ be the height of the fenced-in region. Then the amount of fencing required is $3x + 2y$. Since we can assume the gardener uses all the available fencing material, we get the constraint equation $3x + 2y = 60$. The objective function is the area $A = xy$, which may be expressed, using the constraint equation, as $A = x(30 - 3x/2) = 30x - 3x^2/2$. To maximize the area, we take the derivative $dA/dx = 30 - 3x$, which is zero when $x = 10$. Plugging this back in the constraint equation, we get $y = 15$, so the maximum area is 150 square meters.

5. Let $f(x) = \sin x + \cos x$. Find the absolute minimum and maximum values of $f(x)$ on the interval $[0, \pi]$.
   
   (a) Minimum: $-2$; Maximum: 2
   (b) Minimum: $-\sqrt{2}$; Maximum: $\sqrt{2}$
   (c) Minimum: $-1$; Maximum: 1
   (d) Minimum: $-1$; Maximum: $\sqrt{2}$
   (e) Minimum: $-1$; Maximum: $\sqrt{2}/2$
   (f) Minimum: $-1$; Maximum: 0
   (g) Minimum: $-\sqrt{2}/2$; Maximum: $\sqrt{2}/2$
   (h) Minimum: 0; Maximum: 1
   (i) Minimum: 0; Maximum: $\sqrt{2}$
   (j) Minimum: 0; Maximum: $\sqrt{2}/2$

   (d) $f'(x) = \cos x - \sin x$, which equals zero when $\cos x = \sin x$, so the only critical point in the interval $[0, \pi]$ is $\pi/4$. To find the absolute min and max values, compare the values of $f(x)$ at the critical point and the two endpoints: $f(0) = 1$, $f(\pi/4) = \sqrt{2}$, and $f(\pi) = -1$. So the absolute minimum value is $-1$ and the absolute maximum value is $\sqrt{2}$.

6. A rock is thrown into a pond and causes a circular ripple. If the radius of the ripple is increasing at a rate of 4 feet per second, how fast (in square feet per second) is the area changing when the radius is 10 feet?
   
   (a) $4\pi$
   (b) $8\pi$
   (c) $10\pi$
   (d) $16\pi$
   (e) $40\pi$
   (f) $80\pi$
   (g) $100\pi$
   (h) $200\pi$
   (i) $400\pi$
   (j) $800\pi$

   (f) The relationship between radius and area is $A = \pi r^2$, so $dA/dt = 2\pi r dr/dt$. In this problem, $r = 10$ and $dr/dt = 4$, so $dA/dt = 80\pi$. 
7. The graph of \[ y = \frac{4x^2 + 2x + 1}{x+2} \] has a slant asymptote. What is the equation of the asymptote?

(a) \( y = x - 3 \)
(b) \( y = x + 1 \)
(c) \( y = 2x - 6 \)
(d) \( y = 2x - 1 \)
(e) \( y = 2x + 4 \)
(f) \( y = 4x - 6 \)
(g) \( y = 4x - 1 \)
(h) \( y = 4x + 2 \)
(i) \( y = 6x \)

(f) To find the slant asymptote, long divide. You get \( y = 4x - 6 + \frac{13}{x+2} \), which approaches \( y = 4x - 6 \) as \( x \to \infty \).

8. Which of the following best describes the graph of \( y = x \ln x \) when \( x = 1/e \)?

(a) increasing, concave down
(b) increasing, concave up
(c) decreasing, concave down
(d) decreasing, concave up
(e) local maximum
(f) local minimum
(g) critical point, inflection point
(h) increasing, inflection point
(i) decreasing, inflection point
(j) not differentiable at this point

(f) \( y' = \ln x + 1 \), and \( y'' = 1/x \). When \( x = 1/e \), \( y' = 0 \) and \( y'' > 0 \), so it’s a local minimum.

9. The demand function for mePods is \( p = 200 - x \), where \( p \) is the price (in dollars) at which \( x \) mePods can be sold. The cost (in dollars) of producing \( x \) mePods is \( C(x) = 100 + 20x + 2x^2 \). How many mePods should be produced in order to maximize profit?

(a) 20
(b) 25
(c) 30
(d) 35
(e) 40
(f) 45
(g) 50
(h) 55
(i) 60

(c) Profit is $\Pi(x) = 200x - x^2 - (100 + 20x + 2x^2) = -3x^2 + 180x - 100$. Its derivative is $\Pi'(x) = -6x + 180$, which is zero when $x = 30$.

10. Find the slope of the graph of $2x^2 + y^2 = 3xy$ at the point $(1, 2)$.

(a) $-1/2$
(b) $-1/4$
(c) 0
(d) 2/5
(e) 3/8
(f) 1/4
(g) 1/2
(h) 1
(i) 2
(j) 5/2

(i) Use implicit differentiation with respect to $x$:

$$4x + 2yy' = 3y + 3xy'.$$

You can solve for $y'$ to get

$$y' = \frac{4x - 3y}{3x - 2y},$$

which, when $x = 1$ and $y = 2$, is 2.

11. The half-life of a certain radioactive element is 100 days. How many days will it take for a 100mg sample to decay to 10mg? Choose the closest answer.

(a) 100
(b) 200
(c) 240
(d) 270
(e) 300
(f) 330
(g) 370
(h) 400
(i) 420
(j) 1000
(f) If the half-life is 100 days, then the decay constant is \((\ln 2)/100 = 0.00693\). So we need to solve the following equation for \(t\):

\[
10 = 100e^{-0.00693t}.
\]

The solution is \(t = \frac{\ln(1/10)}{-0.00693}\), which is approximately 330.

12. Find the slope of the graph of \(y = x^{1/x}\) when \(x = 2\). Choose the closest answer.

(a) 0
(b) 0.1
(c) 0.2
(d) 0.3
(e) 0.4
(f) 0.5
(g) 0.6
(h) 0.8
(i) 1
(j) 1.2

(b) This requires logarithmic differentiation. \(\ln y = (1/x) \ln x\), so

\[
\frac{y'}{y} = \frac{-1}{x^2} \ln x + \frac{1}{x^2}.
\]

When \(x = 2\), \(y = \sqrt{2}\), so \(y' = \sqrt{2}(\frac{-1}{4} \ln 2 + \frac{1}{4})\), which is about 0.11.

13. Find the area enclosed by the parabolas \(y = x^2\) and \(y = 8 - x^2\).

(a) 0
(b) 1/3
(c) 1
(d) 4/3
(e) 8/3
(f) 16
(g) 32/3
(h) 64/3
(i) 32

(h) The curves intersect at \(x = \pm 2\), so the area is equal to

\[
\int_{-2}^{2} (8 - 2x^2)dx = [8x - 2x^3/3]_{-2} = 64/3.
\]
14. Suppose that the demand equation for corn is \( p = 200 - x \), where \( p \) is the price (in dollars per ton) at which \( x \) tons of corn are demanded. Suppose that the supply equation is \( p = \frac{x^2}{100} \). If corn is being sold at the equilibrium price, what is the total economic surplus in the corn market? Choose the closest answer.

(a) $1,667  
(b) $3,333  
(c) $5,000  
(d) $6,667  
(e) $8,333  
(f) $10,000  
(g) $11,667  
(h) $13,333  
(i) $15,000  
(j) $20,000

(g) The equilibrium occurs when the supply and demand curves meet: \( 200 - x = \frac{x^2}{100} \) when \( x = 100 \) and \( x = -200 \). In this case, only the positive value, \( x = 100 \), is meaningful. The total surplus is

\[
\int_{0}^{100} (200 - x - \frac{x^2}{100}) \, dx = \left[ 200x - \frac{x^2}{2} - \frac{x^3}{300} \right]_{0}^{100} = 11667.
\]

15. Suppose money is contributed to a retirement account at an annual rate of $5,000. If the rate of return for the investment is 10% compounded continuously, approximately how much will the account be worth after 20 years?

(a) $100,000  
(b) $178,000  
(c) $233,000  
(d) $289,500  
(e) $319,500  
(f) $362,000  
(g) $419,500  
(h) $504,000  
(i) $739,000

(e) The future value is approximated by the integral

\[
\int_{0}^{20} 5000 \, e^{-10t} \, dt = \left[ 5000e^{-10t} \right]_{0}^{20}.
\]

Which is about 319,453.
16. What is \[ \int_0^2 \sqrt{4 - x^2} \, dx \]?

Choose the closest answer.

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5
(f) 6
(g) 7
(h) 8
(i) 9

(c) This integral can be interpreted as the area of a quarter-circle of radius 2. Therefore, the answer is \( \pi \).

17. What is the area below the graph of \( y = \sin x \) between \( x = 0 \) and \( x = \pi \)?

(a) 0
(b) \( \sqrt{2}/2 \)
(c) \( \sqrt{3}/2 \)
(d) 1
(e) \( \sqrt{2} \)
(f) \( \pi/2 \)
(g) \( \sqrt{3} \)
(h) 2
(i) \( \pi \)

(h) \[ \int_0^\pi \sin x \, dx = [-\cos x]_0^\pi = 2. \]

18. Compute \[ \int_1^2 \frac{x^2 + 1}{x} \, dx. \]

Choose the closest answer,

(a) 1
(b) 1.19
(c) 1.38
(d) 2.19
(e) 2.5
(f) 2.65
(g) 2.78
(h) 3.14
(i) 3.33
(j) 3.69

(d)
\[
\int_{1}^{2} \frac{x^{2} + 1}{x} \, dx = \int_{1}^{2} (x + 1/x) \, dx = \left[ \frac{x^{2}}{2} + \ln x \right]_{1}^{2},
\]
which is about 2.19.

19. Use a Riemann sum with \( n = 4 \) and left endpoints to estimate the area under the graph of \( f(x) = e^{-x^2} \) between \( x = 0 \) and \( x = 1 \). Choose the closest answer.

(a) 0.82
(b) 0.99
(c) 1.18
(d) 1.35
(e) 1.51
(f) 1.68
(g) 1.77
(h) 2.13
(i) 3.29

(a) The Riemann sum would be
\[
0.25(e^{0} + e^{-25^2} + e^{-5^2} + e^{-75^2}) \approx .82.
\]

20. Find the function \( f(x) \) for which \( f'(x) = \sqrt{x} \) and \( f(1) = 1 \). What is \( f(4) \)?

(a) 1/4
(b) 3/8
(c) 1
(d) 17/16
(e) 4/3
(f) 2
(g) 5/2
(h) 4
(i) 17/3

(i) The antiderivatives of \( \sqrt{x} \) are of the form \( f(x) = \frac{2x^{3/2}}{3} + C \), and setting \( f(1) = 1 \), we get \( C = 1/3 \). So \( f(x) = \frac{2x^{3/2}}{3} + 1/3 \), and \( f(4) = 17/3 \).