

MATH 128, SPRING 2004
-FINAL EXAM-

Name: _____ WashU ID Number: _____

There are 16 questions worth a total of 100 points. The first 15 questions are worth 6 points each, for a total of 90 points. Question 16 is worth 10 points.

These are all “free response” problems. Write your answers on the test pages. (There are four extra blank pages at the end of the exam in case you need extra space.) Show your work neatly and cross out irrelevant scratchwork, false starts, etc. To get full credit, it must be clear HOW you arrived at your answer.

Please put your name and WashU ID number in the spaces marked above. And please put your Washington University ID number on each of the following pages as they might be separated during grading. Do NOT put your name on these pages.

No calculators with a CAS are allowed.

You might find the following list of Taylor series useful on some of the problems.

(1)

$$\frac{1}{1-x} = 1 + x + x^2 + \dots + x^n + \dots \quad \text{for } -1 < x < 1$$

(2)

$$\frac{1}{1+x} = 1 - x + x^2 - \dots + (-1)^n x^n + \dots \quad \text{for } -1 < x < 1$$

(3)

$$\ln(1-x) = -x - \frac{1}{2}x^2 - \dots - \frac{1}{n}x^n - \dots \quad \text{for } -1 < x < 1$$

(4)

$$\ln(1+x) = x - \frac{1}{2}x^2 + \dots + \frac{(-1)^{n-1}}{n}x^n - \dots \quad \text{for } -1 < x < 1$$

(5)

$$e^x = 1 + x + \frac{1}{2!}x^2 + \dots + \frac{1}{n!}x^n + \dots \quad \text{for } -\infty < x < \infty$$

(6)

$$e^{-x} = 1 - x + \frac{1}{2!}x^2 + \dots + \frac{(-1)^n}{n!}x^n + \dots \quad \text{for } -\infty < x < \infty$$

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1. Find the area bounded by the graphs

$$y = 4 - x^2 \quad \text{and} \quad y = -5$$

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2. Compute

$$\int x e^{x-2} dx$$

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3. Give an example of a function $f(x, y)$ such that $f_x = 3$ and $f_y = 5$.

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4. Find the maximum and minimum of

$$f(x, y) = x^2 - y^2$$

subject to

$$x^2 + y^2 = 25$$

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5. Solve

$$y' = \frac{e^x}{e^y}; \quad y(0) = \ln 2.$$

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6. Solve

$$xy' + 3y = 20x$$

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7. If f is a function which satisfies $f'(x) = x^2 e^x$ and $f(0) = 15$, find the Taylor series at 0 for f . Find the interval of convergence.

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8. A chocolate bar manufacturer is ready to introduce a new low-carb chocolate bar by a national sales campaign. After test marketing the chocolate in St Louis, the market research department estimates that sales (in millions of dollars) will increase at a monthly rate of

$$S'(t) = 10 - 10e^{-0.01t^2} \quad 0 \leq t \leq 20$$

t months after the national campaign is started. Use the fourth degree Taylor polynomial at 0 for $S'(t)$ to approximate total sales during the first five months of the campaign, and estimate the error in this approximation.

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9. Compute

$$\int_{-\infty}^{\infty} \frac{e^x}{(1+e^x)^2} dx$$

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10. Suppose you draw one card from an ordinary well-shuffled deck of 52 cards. If the card is an ace, you win \$3. If it is a king, queen, or jack, you win \$2. If it is a ten, you win \$1. If it is any other card, you lose \$1. Compute the expected value of the amount won.

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11. Let

$$f(x) = \begin{cases} e^{-6x} & \text{if } x \geq 0, \\ 0 & \text{if } x < 0 \end{cases}$$

Find a constant k so that kf is a probability density function for some X , or explain why no such k exists.

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12. Let

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1, \\ 2 - x & \text{if } 1 < x \leq 2, \\ 0 & \text{otherwise} \end{cases}$$

Compute the associated cumulative distribution function.

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13. The number of hours it takes a chimpanzee to learn a new task is a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{4}{9}x^2 - \frac{4}{27}x^3 & \text{if } 0 \leq x \leq 3, \\ 0 & \text{otherwise} \end{cases}$$

What is the expected number of hours it will take a chimpanzee to learn the task?

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14. Scores on a school's entrance exam are normally distributed, with mean 500 and variance 10,000. If the school wishes to admit only the students in the top 20%, what should be the cutoff grade?

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15. If X is an exponentially distributed random variable with median $m = 4 \ln 2$ and mean μ , find

$$P(X \leq \mu).$$

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16. A certain gas station sells X thousand gallons of gas each week. Suppose that the cumulative distribution function for X is

$$F(x) = 1 - \frac{1}{4}(2 - x)^2, \quad 0 \leq x \leq 2$$

- (1) If the tank contains 1.6 thousand gallons of gas at the beginning of the week, find the probability that the gas station will have enough gas for its customers throughout the week.
- (2) How much gas must be in the tank at the beginning of the week in order to have a probability of 0.99 that there will be enough gasoline for the week?
- (3) Compute the probability density function for X .

