Math 128 Exam 1 September 17, 2003

This exam has 20 questions. For questions 1 through 18 indicate your answer on the answer card. Problems 19 and 20 will be graded by hand. For those two questions you must indicate work which justifies your answer to receive full credit.

For answers in dollars, round to the nearest dollar

1.

$$f(x) = x^2 - 2\sqrt{x} + \frac{3}{x^3}$$

Find $f'(1)$.

$$f'(x) = 2x - \frac{1}{\sqrt{x}} - \frac{9}{x^4}$$

$$f'(1) = 2 - 1 - 9 = -8$$

- **a.** $\boxed{-8}$
- **b.** -6
- **c.** -4
- **d.** -2
- **e.** 0
- **f.** 2
- **g.** 4
- **h.** 6
- **i.** 8
- **j.** 10

2. Consider the function $f(x) = 2x^3 - 9x^2 + 12x - 8$. On which interval(s) is the function decreasing ?

a. $(2.914, \infty)$

b. $(-\infty, 2.914)$

c. It is never decreasing.

d. It is always decreasing.

e. $(-\infty, 1)$

f. $(-\infty, 2)$

g. $(-\infty, 1)$ and $(2, \infty)$

h. $(1,2)$

i. $(3/2, \infty)$

j. $(-\infty, 3/2)$

$f'(x) = 6x^2 - 18x + 12$
$= 6(x^2 - 3x + 2)$
$= 6(x-1)(x-2)$

for $x < 1$ $\quad f'(x) = 6 \cdot neg \cdot neg > 0$

$1 < x < 2$ $\quad f' = 6 \cdot pos \cdot neg < 0$

$x > 2$ $\quad f' = 6 \cdot pos \cdot pos$

3. Find the absolute maximum, if there is one, of the function $x^2 e^{-x/4}$ on the interval $[0,\infty)$.

a. 0

b. $e^{1/4}$

c. $4e^{-1/2}$

d. $9e^{-3/4}$

e. $16e^{-1}$

f. $25e^{-5/4}$

g. $36e^{-3/2}$

h. $49e^{-7/4}$

i. $64e^{-2}$

j. There is no absolute maximum.

$f'(x) = 2xe^{-x/4} - \frac{1}{4}x^2 e^{-x/4}$

$= \left(2x - \frac{x^2}{4}\right) e^{-x/4}$

so $f'(x) = 0$ when $\left(2x - \frac{x^2}{4}\right) = 0$, $x = 0, 8$

0 f↑ 8 f↓

so $x = 8$ is the max

$f(8) = 64e^{-8/4}$

4. Evaluate the integral

$$\int (x^3 - 3e^x + \tfrac{3}{x})dx.$$

a. $3x^2 - 3e^x + 3\ln x$

b. $\frac{1}{4}x^4 - \frac{1}{3}e^x + 3\ln x$

c. $3x^2 - 3e^x - 3/x^2$

$\boxed{\textbf{d. } \frac{1}{4}x^4 - 3e^x + 3\ln x}$

e. ~~$x^4 - 3e^x + 3/x$~~

f. $3x^3 + 3e^x - 3\ln x$

g. $\frac{1}{4}x^4 + e^x - 3\ln x$

h. $\frac{1}{4}x^4 + 3e^x + 3/x$

i. $\frac{1}{4}x^4 - e^x - 3/x$

j. $\frac{1}{4}x^4 - 3e^x + 3/x$

$$\int x^3 - 3e^x + \frac{3}{x}\,dx$$

$$= \frac{x^4}{4} - 3e^x + 3\ln x$$

5. Evaluate the integral

$$\int_1^2 \frac{1}{3x+2}dx.$$

a. 14/3

b. $\ln 14 - \ln 3$

c. 3/13

d. 8/5

e. 24/5

f. 5/24

g. $3(\ln 24 - \ln 5)$

$\boxed{\textbf{h. } \frac{1}{3}(\ln 8 - \ln 5)}$

i. $3(\ln 8 - \ln 5)$

j. $24\ln 5$

$$\int \frac{1}{3x+2}\,dx = \frac{1}{3}\ln(3x+2) \quad (\text{e.g by } u = 3x+2)$$

so $\text{Ans} = \frac{1}{3}\ln(3x+2)\Big|_1^2$

$$= \frac{1}{3}\ln 8 - \frac{1}{3}\ln 5$$

6. Evaluate the integral

$$\int_0^1 x^2 e^{x^3+1}\,dx.$$

a. e^2

b. $e^2 - 3$

c. $2(e^2 - 3)$

d. $2(e^2 - e)$

e. $(e^2 - e)/2$

f. $\boxed{(e^2 - e)/3}$

g. $3e^2 - e$

h. $e^4 - 3e^2$

i. $3e^4 - e$

j. $3(e^4 - e)$

$\int x^2 e^{x^3+1}\,dx$

$u = x^3+1$

$du = 3x^2\,dx$

$\int x^2 e^{x^3+1}\,dx = \int \frac{1}{3} e^{x^3+1} 3x^2\,dx$

$= \int \frac{1}{3} e^u\,du$

$= \frac{1}{3} e^u$

$= \frac{1}{3} e^{x^3+1}$

Ans $= \frac{1}{3} e^{x^3+1} \Big|_0^1 = \frac{1}{3}e^2 - \frac{1}{3}e$

7. Find the area between the curves $y = x^2 + 3$ and $y = 4x$.

a. 3/4
b. 5/4
c. 3/8
d. 5/8
e. 8/3
f. 4/3
g. 7/3
h. 4/5
i. 7/4
j. 4/7

intersection at $4x = x^2 + 3$

so $x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 1, 3$

between $x = 1$ + $x = 3$ $4x$ is larger (e.g. test at $x = 2$)

$$\text{Area} = \int_1^3 4x - (x^2+3)\, dx$$

$$= 2x^2 - \frac{x^3}{3} - 3x \Big|_1^3$$

$$= 18 - 9 - 9 - (2 - \tfrac{1}{3} - 3)$$

$$= 4/3$$

8. The Lorenz curve for income distribution is a certain country is given by $y = xe^{x-1}$. Find the associated Gini index.

a. $e/3$
b. $(e+1)/3$
c. $1 - 2/e$.
d. $1/4 - e^{-1}/2$
e. $(e^2 - e)/3$
f. $e^{-1} + e^{-2}$
g. $\boxed{1 - 2e^{-1}}$
h. 2/5
i. 2/3
j. 3/4

$$\text{Index} = 2\int_0^1 x - xe^{x-1}\, dx$$

$$= 2\int_0^1 x\, dx - 2\int_0^1 xe^{x-1}\, dx$$

$$= 1 - 2e^{-1}\int_0^1 xe^x\, dx$$

$$\int xe^x\, dx = xe^x - e^x \qquad \left(\begin{array}{l}\text{IBP} \\ u = x \\ dv = e^x dx\end{array}\right)$$

$$\text{so } \int_0^1 xe^x\, dx = xe^x - e^x \Big|_0^1$$

$$= (e - e) - (0 - 1) = 1$$

$$\text{so ans} = 1 - 2e^{-1} \cdot 1 = 1 - 2e^{-1}$$

9. Suppose you invest $1,000 at 6% compound interest. How long will it be before the value of the investment is $3,000?

a. 15.65 years
b. 15.78 years
c. 16.23 years
d. 16.68 years
e. 16.93 years
f. 17.12 years
g. 17.44 years
h. 18.02 years
i. 18.31 years
j. 19.23 years

$A(t) = Pe^{rt}$

$3000 = 1000e^{.06t}$

$3 = e^{.06t}$

$\ln 3 = .06t$

$t = \frac{\ln 3}{.06} = 18.31$

10. If you invest $2,000 at 5.5% compound interest for $2\frac{1}{2}$ years, how much will the investment be worth then?

a. $2275
b. $2280
c. $2287
d. $2292
e. $2295
f. $2301
g. $2307
h. $2313
i. $2318
j. $2323

$A(t) = Pe^{rt}$

$= 2000\,e^{(.055)(2.5)}$

$= \$2{,}294.80$

11. What is the total income produced by the income stream $f(t) = 1000e^{.2t}$ in the first three years?

a. \$3995

b. \$4004

c. \$4011

d. \$4025

e. \$4031

f. \$4044

g. \$4051

h. \$4076

i. \$4097

j. \$4111

$$\text{Income} = \int_0^3 f(t)\,dt$$

$$= \int_0^3 1000\, e^{.2t}\,dt$$

$$= \frac{1000}{.2} e^{.2t}\Big|_0^3$$

$$= \frac{1000}{.2}\left(e^{.6} - 1\right)$$

$$= \$4{,}110.59$$

i. $\$13,832$

$e^{.6}(1000)\left(\frac{1}{\ }\right)(e \ -1)$

15. What is the present value of that income stream?

a. \$3,702

b. \$3,735

c. \$3,755

d. \$3,786

e. \$3,797

f. \$9,611

g. \$8,821

h. \$8,768

i. \$8,314

$\boxed{\textbf{j. } \$7,520}$

$PV = \int_0^{10} 1000\, e^{-.06t}\,dt$

$= \text{as on \# 12}$

$= 1000\left(\frac{-1}{.06}\right)\left(e^{-.6} - 1\right)$

$= \$7,519.80$

16. Evaluate

$$\int_1^2 2xe^{-x}dx$$

a. 1/3

b. 2/3

c. 1

d. $e^{-2} - e^{-1}$

e. $-2e^{-2} + 3e^{-1}$

f. $4e^{-2} - 2e^{-1} - 1$

g. $e^{-2} - 2e^{-1}$

$\boxed{\textbf{h. } -6e^{-2} + 4e^{-1}}$

i. $4e^{-2} - 2e^{-1} - 1$

j. $4e^{-2} + 2e^{-1} - 1$

$\int 2xe^{-x}dx = 2\int xe^{-x}dx$

$u = x \qquad du = 1$

$dv = e^{-x}dx \qquad v = -e^{-x}$

$\int = 2\left[x(-e^{-x}) - \int -e^{-x}dx\right]$

$= -2xe^{-x} + 2\int e^{-x}dx$

$= -2xe^{-x} - 2e^{-x}$

$\text{Ans} = -2xe^{-x} - 2e^{-x}\Big|_1^2$

$= -4e^{-2} - 2e^{-2} - (-2e^{-1} - 2e^{-1})$

$= -6e^{-2} + 4e^{-1}$

Name:______________________ ID Number:______________________

This question will be hand graded, to get full credit in needs to be clear HOW you arrived at your answer.

21. Find the area between $f(x) = x - x^2$ and $g(x) = -2x$ by

a. Sketching both functions on the given coordinate set and labeling which is which.

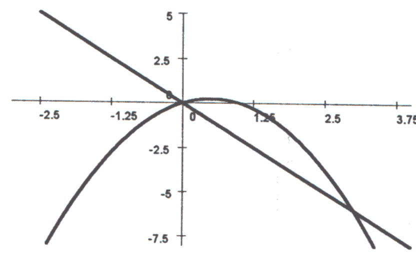
Cross when

$x - x^2 = -2x$

$3x - x^2 = 0$

$x(3 - x) = 0$

$x = 0, 3$



b. Expressing the area as an integral or combination of integrals.

$$\text{Ans} = \int_0^3 x - x^2 - (-2x)\, dx$$

c. Explaining briefly but clearly with reference to the graph what geometric principles you used in determining how to set up the integral(s)

$$\text{Area} = \int_0^3 \text{top} - \text{bottom}\, dx$$

d. Evaluate the integral(s).

$$\int_0^3 3x - x^2\, dx = \frac{3x^2}{2} - \frac{x^3}{3} \Big|_0^3$$

$$= \frac{27}{2} - \frac{27}{3} = 27\left(\frac{1}{2} - \frac{1}{3}\right)$$

$$= \frac{27}{6} = \frac{9}{2}$$

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20. Here is the data on how much of the family income goes to five different groups in Country A:

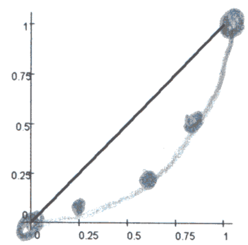
Bottom 20%	Next 20%	Mid 20%	Next 20%	Top 20%
2%	8%	15%	25%	50%

a. On the given grid, which already has the line $x = y$ included for your convenience, sketch the Lorenz Curve for the data.

we need the cumulative numbers

Bottom .2 .4 .6 .8 1

Fraction .02 .1 .25 .5 1



The Lorenz curve.

b. Expain briefly but clearly what a meant by a right sum for estimating the area between the Lorenz curve.

estimate the area for each range of x; $0<x<.2, \ldots, .8<x<1$ by ~~w~~ length (=.2) times hight of graph at right edge of interval — Then add the five contributions

c. In fact the right sum for this data is.374 and the left sum is .174. Using these numbers compute the Gini index associated with this curve.

estimate of area $= \frac{1}{2}(.174 + .374) = .274$

Index $= (.5 - .274)/.5 = 1 - .548 = .452$

d. Suppose the Lorenz curve for Country B is farther from the diagonal line:

i. Will Country B have a larger or smaller Gini index than Country A?

larger

ii. Is income in Country B more evenly distributed than in Country A?

NO