Calculus II for the Life, Social and Managerial Sciences

Math 128 — Fall 2007

In-term exam September 26

Name: ___________________________ Student-ID: ___________________________

This exam contains sixteen problems. Problems 1 - 14 are multiple choice problems, which each count 5% towards your total score. Problem 15 and 16 will be hand-graded (with a possibility of partial credit) and count 15% each towards your total score.

Problem 1

\[ \int (e^{2x} + 4x^3 + 7) \, dx = \]

A) \( \frac{1}{2} e^{2x} + x^4 + C \)
B) \( \frac{1}{2} e^{2x} + 4x^4 + 7x + C \)
C) \( \frac{1}{2} e^{2x} + x^4 + 7x + C \)
D) \( e^{2x} + x^4 + 7x + C \)
E) \( e^{2x} + 4x^4 + 7x + C \)
F) \( e^{2x} + x^4 + 7 + C \)
G) \( 2e^{2x} + 12x^2 + C \)
H) \( 2e^{2x} + x^4 + 7x + C \)
Problem 2

Suppose that the rate (in tons per year) at which pollutants are discharged into a lake is given by

\[ R(t) = \frac{1000}{(1 + t)^2} \]

where \( t \) is the time in years since January 1, 2000. Find the total amount of discharged pollutants (in tons) between January 1, 2000 and January 1, 2004.

A) 300
B) 400
C) 500
D) 600
E) 700
F) 800
G) 900
H) 1000
Problem 3

Find the area under the curve $y = x^9 + 25x + 7$ from $x = 0$ to $x = 1$.

A) 20.1  
B) 20.2  
C) 20.3  
D) 20.4  
E) 20.5  
F) 20.6  
G) 20.7  
H) 20.8
Problem 4

Determine the average value of \( f(x) = 200e^{-4x} \) over the interval from \( x = 0 \) to \( x = 2 \).

A) \( 25 - 25e^{-4} \)
B) \( 25 - 25e^{-8} \)
C) \( 50 - 50e^{-4} \)
D) \( 50 - 50e^{-8} \)
E) \( 100 - 100e^{-4} \)
F) \( 100 - 100e^{-8} \)
G) \( 200 - 200e^{-4} \)
H) \( 200 - 200e^{-8} \)
Problem 5

Find the consumers’ surplus for the demand curve $p = \frac{4000}{(x+10)^2} - 3$ at the sales level $x = 10$.

A) 50  
B) 100  
C) 150  
D) 200  
E) 250  
F) 300  
G) 350  
H) 400
Problem 6

Suppose that money is deposited daily into a savings account at an annual rate of $2000. If the account pays 5% interest compounded continuously, estimate (to the nearest dollar) the balance in the account at the end of 4 years.

A) $8856
B) $8861
C) $8868
D) $8875
E) $8882
F) $8889
G) $8896
H) $8903
Problem 7

Find the volume of the solid of revolution obtained from revolving the region below the graph of \( y = \sqrt{x} \) from \( x = 3 \) to \( x = 5 \) about the \( x \)-axis.

A) \( \pi \)
B) \( 2\pi \)
C) \( 3\pi \)
D) \( 4\pi \)
E) \( 5\pi \)
F) \( 6\pi \)
G) \( 7\pi \)
H) \( 8\pi \)
Problem 8

Which function \( f(x, y) \) do these level curves belong to?

A) \( f(x, y) = y + (x - 1)^2 \)
B) \( f(x, y) = (x - 1)^2 + y^2 \)
C) \( f(x, y) = 2(x - 1)^2 + y^2 \)
D) \( f(x, y) = y + x - 1 \)
E) \( f(x, y) = (x - 1)^2 + 2y^2 \)
F) \( f(x, y) = y - (x - 1)^2 \)
G) \( f(x, y) = y - (x + 1)^2 \)
H) \( f(x, y) = y - x^2 \)
Problem 9

Compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of the function

$$f(x, y) = e^{3x} + 5(x - y)^3$$

Then $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = \cdots$

A) $e^{3x} + 15(x - y)^2$
B) $3e^{3x} + 30(x - y)^2$
C) $3xe^{3x} + 30(x - y)^2$
D) $3e^{3x}$
E) $e^{3x}$
F) $e^{3x} + 30(x - y)^2$
G) $3e^{3x} + 15(x - y)^2$
H) $3e^{3x} - 30(x - y)^2$
Problem 10

Let \( f(x, y, z) = x^2 y - 3xz + 6y^3z + 2y - 7z^2 + 37 \).

Evaluate \( \frac{\partial f}{\partial z} \) at the point \((14, 1, -3)\).

A) 1
B) -3
C) 6
D) 2
E) -7
F) 37
G) 14
H) -78
Problem 11

Compute the second partial derivatives $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ of the function

$$f(x, y) = xe^y + 2x^2y^2$$

Then $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \cdots$

A) $xe^y + 4y^2$
B) $xe^y$
C) $4y^2$
D) $xe^y + 4x^2$
E) $xe^y + 4x^2 + 4y^2$
F) $4x^2 + 4y^2$
G) $4x^2$
H) 0
Problem 12

Using Lagrange multipliers, minimize \( f(x, y) = (x - 1)^2 + y^2 \), subject to the constraint \( x + 2y - 11 = 0 \).

At which point \( (x, y) \) does this minimum occur?

A) \( (3, 4) \)
B) \( (3, -4) \)
C) \( (-4, 3) \)
D) \( (-4, -3) \)
E) \( (-3, -4) \)
F) \( (-3, 4) \)
G) \( (4, -3) \)
H) \( (4, 3) \)
Problem 13

Use partial derivatives to obtain the formula for the best least-squares fit to the data points (1, 8), (2, 4), (4, 3).

This line is given by which formula?

A) \( y = -0.5x + 8.5 \)
B) \( y = -x + 7.5 \)
C) \( y = -x + 8.5 \)
D) \( y = -1.5x + 7.5 \)
E) \( y = -1.5x + 7 \)
F) \( y = -x + 7 \)
G) \( y = -1.5x + 8.5 \)
H) \( y = -0.5x + 8 \)
Problem 14

Evaluate the iterated integral

\[ \int_0^1 \left( \int_1^2 (6xy + 2) \, dy \right) \, dx \]

A) 4
B) 4.5
C) 5
D) 5.5
E) 6
F) 6.5
G) 7
H) 7.5
Written problem - Show your work

Problem 16

Find all points where \( f(x, y) = x^3 + y^2 - 3x - 8y + 12 \) has a possible extreme value. Then, using the second-derivative test, determine for each of these points its nature (relative maximum, relative minimum, or saddle point).
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Written problem - Show your work

Problem 15

The graphs of \( f(x) = x^2 - 4x \) and \( g(x) = 2x - 8 \) intersect at the points \((2, -4)\) and \((4, 0)\). Calculate the area between these two graphs.