Calculus II for the Life, Social and Managerial Sciences

Math 128 — Fall 2007

In-term exam November 14

Name: Student-ID:

This exam contains sixteen problems. Problems 1 – 14 are multiple choice problems, which each count 5% towards your total score. Problem 15 and 16 will be hand-graded (with a possibility of partial credit) and count 15% each towards your total score.

Problem 1

Which function $y(t)$ is a solution to the following initial value problem?

$$y' - 2y = 0, \quad y(0) = 5$$

A) $y(t) = e^{2t}$
B) $y(t) = e^{5t}$
C) $y(t) = e^{-2t}$
D) $y(t) = e^{-5t}$
E) $y(t) = 5e^{2t}$
F) $y(t) = 2e^{5t}$
G) $y(t) = 5e^{-2t}$
H) $y(t) = 2e^{-5t}$
Problem 2

Find the constant solution of the following differential equation.

\[ y' = 3t^2(y + 2)^2 \]

A) \( y = -2 \)
B) \( y = -\sqrt{2} \)
C) \( y = -1 \)
D) \( y = 0 \)
E) \( y = 1 \)
F) \( y = \sqrt{2} \)
G) \( y = 2 \)
H) \( y = 3 \)
Problem 3

Find all other solutions to the differential equation in Problem 2 by actually solving the equation.

A) \( y(t) = \frac{1}{2} + C \)
B) \( y(t) = \frac{1}{2t+C} + 2 \)
C) \( y(t) = \frac{1}{2} + C \)
D) \( y(t) = \frac{1}{2t+C} - 2 \)
E) \( y(t) = \frac{3}{2} + C \)
F) \( y(t) = \frac{1}{2t+C} + 2 \)
G) \( y(t) = -\frac{3}{2} + C \)
H) \( y(t) = -\frac{1}{2t+C} - 2 \)
Problem 4

Solve the initial value problem

\[ y' + 2y = e^{-t}, \quad y(0) = 4 \]

A) \( y(t) = e^{-t} + 3e^{-2t} \)
B) \( y(t) = 3e^{-t} + e^{-2t} \)
C) \( y(t) = e^{-t} - 3e^{-2t} \)
D) \( y(t) = 3e^{-t} - e^{-2t} \)
E) \( y(t) = -e^{-t} + 3e^{-2t} \)
F) \( y(t) = -3e^{-t} + e^{-2t} \)
G) \( y(t) = -e^{-t} - 3e^{-2t} \)
H) \( y(t) = -3e^{-t} - e^{-2t} \)
Problem 5

An initial deposit of $1,000 is made into an account earning 8% on a yearly basis, compounded continuously. Money is then continuously withdrawn at a constant rate of $100 a year, until the account is depleted. Which initial value problem models this situation?

A) $y' = 0.8y + 100, \quad y(0) = 1000$

B) $y' = 0.8y + 1000, \quad y(0) = 100$

C) $y' = 0.08y + 100, \quad y(0) = 1000$

D) $y' = -0.08y + 100, \quad y(0) = 1000$

E) $y' = 0.08y - 100, \quad y(0) = 1000$

F) $y' = 0.08y - 1000, \quad y(0) = 100$

G) $y' = 0.8y - 100, \quad y(0) = 0$

H) $y' = 0.8y - 100, \quad y(0) = 1000$
Problem 6

Suppose that a fish population in a lake develops according to the logistic equation

\[ N'(t) = 0.1N - 0.0001N^2 \]

where \( t \) is measured in weeks and \( N(0) = 100 \).

Determine the carrying capacity (\( K \)), the intrinsic rate of growth (\( r \)) and the size of the population when \( N'(t) \) reaches a maximum value (let’s call this size \( M \)).

A) \( K = 1000, \ r = 0.1, \ M = 500 \)
B) \( K = 1000, \ r = 0.1, \ M = 100 \)
C) \( K = 1000, \ r = 0.0001, \ M = 500 \)
D) \( K = 1000, \ r = 0.0001, \ M = 100 \)
E) \( K = 2000, \ r = 0.1, \ M = 500 \)
F) \( K = 2000, \ r = 0.1, \ M = 100 \)
G) \( K = 2000, \ r = 0.0001, \ M = 500 \)
H) \( K = 2000, \ r = 0.0001, \ M = 100 \)
Problem 7

Use Euler's method with $n = 2$ on the interval $2 \leq t \leq 3$ to approximate the solution $y(t)$ to the initial value problem $y' = t - 2y$, $y(2) = 3$. Give as your answer the approximation of $y(3)$.

A) $y(3) \approx 1.05$
B) $y(3) \approx 1.15$
C) $y(3) \approx 1.25$
D) $y(3) \approx 1.35$
E) $y(3) \approx 1.45$
F) $y(3) \approx 1.55$
G) $y(3) \approx 1.65$
H) $y(3) \approx 1.75$
Problem 8

Evaluate

\[ \int_{0}^{1} \frac{2x}{(2 + x^2)^2} \, dx \]

A) \( \frac{1}{12} \)
B) \( \frac{1}{6} \)
C) \( \frac{1}{4} \)
D) \( \frac{1}{3} \)
E) \( \frac{1}{2} \)
F) \( \frac{2}{3} \)
G) \( \frac{3}{4} \)
H) \( \frac{5}{6} \)
Problem 9

Find the first Taylor polynomial of \( f(x) = \sqrt{x} \) at \( x = 2 \), and use it to approximate \( \sqrt{2.3} \).

A) 1.5103
B) 1.5123
C) 1.5143
D) 1.5163
E) 1.5183
F) 1.5203
G) 1.5223
H) 1.5243
Problem 10

Compute the second Taylor polynomial of \( f(x) = e^{x^2} \) at \( x = 0 \), and use it to approximate the area under the graph of \( f \) from \( x = 0 \) to \( x = 1 \).

\[
\int_0^1 e^{x^2} \, dx \approx \ldots
\]

A) \( \frac{3}{5} \)
B) \( \frac{2}{3} \)
C) \( \frac{3}{4} \)
D) \( \frac{4}{5} \)
E) \( \frac{5}{4} \)
F) \( \frac{4}{3} \)
G) \( \frac{3}{2} \)
H) \( \frac{5}{9} \)
Problem 11

Find the third Taylor polynomial of \( f(x) = x^4 \) at \( x = 2 \).

A) \( 16 + 32(x - 2) + 24(x - 2)^2 + 8(x - 2)^3 \)
B) \( 16 + 8(x - 2) + 32(x - 2)^2 + 24(x - 2)^3 \)
C) \( 16 + 24(x - 2) + 8(x - 2)^2 + 32(x - 2)^3 \)
D) \( 16 + 32(x - 2) + 8(x - 2)^2 + 24(x - 2)^3 \)
E) \( 16 + 24(x - 2) + 32(x - 2)^2 + 8(x - 2)^3 \)
F) \( 16 + 8(x - 2) + 24(x - 2)^2 + 32(x - 2)^3 \)
G) \( 32 + 16(x - 2) + 8(x - 2)^2 + 24(x - 2)^3 \)
H) \( 32 + 24(x - 2) + 16(x - 2)^2 + 8(x - 2)^3 \)
Problem 12

Evaluate

\[ \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin(x - 2x) \, dx \]

A) 0  
B) \frac{1}{6}  
C) \frac{1}{6}  
D) \frac{1}{4}  
E) \frac{1}{6}  
F) \frac{1}{2}  
G) 1  
H) 2
Problem 13

Find the sum of the geometric series $5 + 4 + 3.2 + 2.56 + 2.048 + \cdots$, if it exists.

A) 5
B) 10
C) 15
D) 20
E) 25
F) 30
G) 35
H) This series diverges.
Problem 14

What is the Taylor series for \( f(x) = 2x\left(\frac{1}{1-x} + 1\right)\)?

A) \(2x^2 + 2x^3 + 2x^4 + 2x^5 + \cdots\)
B) \(4x + 2x^2 + 2x^3 + 2x^4 + \cdots\)
C) \(2x + 2x^2 + 2x^3 + 2x^4 + \cdots\)
D) \(4 + 2x + 2x^2 + 2x^3 + \cdots\)
E) \(2x + 2x^2 + 2x^3 + 2x^4 + \cdots\)
F) \(2x + 4x^2 + 4x^3 + 4x^4 + \cdots\)
G) \(4x + 2x^2 + 4x^3 + 2x^4 + \cdots\)
H) \(2x + 4x^2 + 2x^3 + 4x^4 + \cdots\)
Problem 15
Solve the initial value problem
\[ y' - 2ty = -2t, \quad y(0) = 7 \]
Written problem - Show your work

Problem 16

Find the third Taylor polynomial of \( f(x) = \sin(3x) \) at \( x = 0 \) (call it \( p_3(x) \)).