Math 128  Exam 3  Spring ’10

You may use a (non-programmable) scientific calculator and a 3 × 5 note card for the exam. This exam has 22 questions. The first 8 are True-False type questions worth two points each. The next 14 are multiple choice questions worth 6 points each.

Part I

Some of the statements below are always correct the others are sometimes incorrect. Indicate which are which by marking

[ ] for statements that are [ ] always true
[ ] for statements that may [ ] be false

1. If the maximum value of the function \( f(x, y) \) is 0 then the minimum of the function \( e^{-f(x,y)} \) is 1.

2. Suppose profit, \( \pi \), is a function of two variables, \( K, L \) and that you have selected the values of \( K \) and \( L \) which maximize \( \pi \). The shadow price of \( K \) tells, approximately, how much more of input \( L \) you would need to maintain the same level of profit if the amount of input \( K \) were reduced one unit.

3. Given five data points \( (x_1, y_1), \ldots, (x_5, y_5) \), the regression line for that set of data is the polygonal line connecting those five points.

4. General theory insures that the function \( f(x, y) = x^2 + xy^3 + e^{xy} \) will have a maximum value on the region of \( (x, y) \) with \( x^2 + 4y^4 \leq 6 \) and that that maximum will occur at a point where

\[
\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0.
\]

5. The function \( f \) is defined for all \( (x, y) \). If at a critical point of \( f(x, y) \) we have \( f_{xx} = f_{yy} = 1 \) and \( f_{xy} = 0 \) then that point is a local minimum for the function but it may not be the actual minimum.

6. Suppose that you plan to use Lagrange multipliers to find the point on both of the surfaces

\[
x^2 + 2y^2 = 10
\]
\[
x^2 - y^2 + z^2 = 100
\]

which is closest to the point \((1, 2, 3)\). You will need to solve a system of 5 equations in 5 unknowns.

7. The method of Lagrange multipliers produces candidate points for maximizing \( f(x, y) \) given the constraint \( g(x, y) = c \). The geometric idea underlying the method is that if \( f(x_0, y_0) = A \) is the maximum then at the point \( (x_0, y_0) \) the curve \( f(x, y) = A \) and the curve \( g(x, y) = c \) cross at right angles.

8. The maximum value of \( f(x) = x^a e^{-x} \), \( x > 0 \) is a number that depends on \( a \). The Envelope Theorem can be used to study how that maximum depends on \( a \).
Part II: Multiple Choice

9. Find the points, if any, where the function $f(x,y) = 12x^2 + 6y^2 + 12xy - 6x + 2$ might attain its maximum value and its minimum value.

   a. No maximum, no minimum.
   b. Maximum at $(1/2, -1/2)$, no minimum.
   c. No maximum, minimum at $(1/2, 1/2)$
   d. Maximum at $(1, 0)$, no minimum.
   e. No maximum, minimum at $(1/2, -1/2)$
   f. Maximum at $(0, -1)$, minimum at $(1/2, 1/2)$
   g. No maximum, minimum at $(1/2, 0)$
   h. Maximum at $(1/2, -1/2)$, no minimum
   i. Maximum at $(1, -1/2)$, minimum at $(1, 1/2)$
   j. No maximum, minimum at $(1, 2)$

10. Find the points, if any, where the function

$$f(x,y) = \frac{6x}{1+x^2} + 7y + 5$$

might attain its maximum value and its minimum value.

   a. No maximum, no minimum.
   b. Maximum at $(-1/3, 1)$, no minimum.
   c. No maximum, minimum at $(-2, -4)$
   d. Maximum at $(1, 0)$, no minimum.
   e. No maximum, minimum at $(1/2, 1)$
   f. Maximum at $(0, -1)$, minimum at $(1/2, 1/3)$
   g. Maximum at $(1, 0)$, minimum at $(1/2, 0)$
   h. Maximum at $(1, 1/3)$, minimum at $(0, 0)$
   i. Maximum at $(1, -1/2)$, minimum at $(1, 1/2)$
   j. Maximum at $(0, -1)$, minimum at $(1, 2)$
11. Find the points, if any, where the function \( f(x, y) = 4x + 8y + 4xy + x^3 + 1 \) might attain its maximum value and its minimum value.

   a. No maximum, no minimum.
   b. Maximum at \((-2, 1)\), no minimum.
   c. No maximum, minimum at \((2, 4)\)
   d. Maximum at \((-2, 4)\), no minimum.
   e. No maximum, minimum at \((1, 1)\)
   f. Maximum at \((0, -1)\), minimum at \((4, -2)\)
   g. Maximum at \((1, 0)\), minimum at \((0, 0)\)
   h. Maximum at \((0, 0)\), minimum at \((1, 0)\)
   i. Maximum at \((1, -4)\), minimum at \((-4, 2)\)
   j. Maximum at \((0, 4)\), minimum at \((1, 2)\)

12. The function \( f(x, y) = x^2 + 2xy^2 + 2y^2 \) has three critical points. Locate them and classify the type of critical points they are. When listed from top to bottom (i.e., in order of decreasing \( y \) coordinate) they are

   a. A saddle point, a local maximum, a local minimum.
   b. A saddle point, a local minimum, a local maximum.
   c. A local minimum, a local minimum, a local minimum.
   d. A local maximum, a saddle point, a local minimum.
   e. A local minimum, a saddle point, a local maximum.
   f. A saddle point, a local minimum, a saddle point.
   g. A saddle point, a saddle point, a saddle point.
   h. A local maximum, a local minimum, a local minimum.
   i. A local maximum, a local maximum, a local maximum.
   j. A local maximum, a local minimum, a saddle point.
13. The function
\[ f(x,y,z) = 3 - x^2 - 2y^2 - 3z^2 - 2xz - 2yz \]
has a maximum. What is the maximum value?

a. 3
b. 2
c. 1
d. 0
e. 1
f. 2
g. 3
h. 6
i. 12
j. 18

14. Suppose that with the constraint \( g(x,y) = 10 \), the function \( f(x,y) \) has minimum 2 and no maximum. With the same constraint what can you say about the extreme values of the function \( h(x,y) \) defined by
\[ h(x,y) = \frac{1}{f(x,y)} \]?

a. minimum 2 and no maximum
b. maximum 2 and no minimum
c. minimum 2 and can’t tell about the maximum
d. maximum 2 and can’t tell about the minimum
e. can’t tell about the maximum or minimum
f. minimum 1/2 and no maximum
g. maximum 1/2 and no minimum
h. minimum 1/2 and can’t tell about the maximum
i. maximum 1/2 and can’t tell about the minimum
j. maximum 2 and minimum 1/2
15. Find the points that are candidates for solving the constrained optimization problem
maximize \( x^2y \) with \( x + 2y = 6 \)

a. (0,3)  
b. (0,3),(3,1)  
c. (0,2),(1,4),(0,1)  
d. (1,0),(0,1)  
e. (2,3),(2,1),(0,0)  
f. (4,1)  
g. (1,1),(2,1)  
h. (0,0),(1,1)  
i. (0,3),(4,1)  
j. (0,2),(2,1)

16. Find the points that are candidates for solving the constrained optimization problem
minimize \( 2x + 3y \) with \( x > 0 \) and \( xy = 6 \)

a. (1,2)  
b. (2,3)  
c. (1,3)  
d. (2,1)  
e. (3,1)  
f. (3,2)  
g. (1,1)  
h. (2,2)  
i. (3,3)  
j. (1,1),(2,2)
17. Find the points that are candidates for solving the constrained optimization problem

\[
\text{maximize } 3x + 4y \text{ with } \sqrt{x} + 2y = 1
\]

a. \((0, 1/2)\)
b. \((1, 0)\)
c. \((1/9, 1/3)\)
d. \((4, -1/2)\)
e. \((0, 1/2), (1, 0)\)
f. \((1, 0), (4, -1/2)\)
g. \((1/9, 1/3), (1, 0)\)
h. \((4, -1/2), (0, 1/2)\)
i. \((0, 1/2), (1/9, 1/3)\)
j. \((4, 1/2), (1/9, 1/3)\)

18. The profit from a manufacturing process using total resources \(R\) is \(xy\) where \(2x + 3y = R\). What is the shadow price of resources when \(R = 8\)?

a. \(1/3\)
b. \(1/2\)
c. \(2/3\)
d. \(3/4\)
e. \(1\)
f. \(3/2\)
g. \(2\)
h. \(3\)
i. \(4\)
j. cannot be determined from the given information
19. What is the point on the plane $2x + 4y + 6z = 28$ closest to the origin?

a. (9, 5, 4)
b. (7, 5, 5),
c. (5, 5, 6)
d. (0, 11, 4)
e. (−1, 0, 5)
f. (0, 1, 4)
g. (1, 2, 3)
h. (2, 3, 2)
i. (3, 4, 1)
j. (4, 5, 0)

20. Find the maximum and minimum, if there are such, of

$$x^2 + y^2 + 1 \text{ constrained by } x^2 - xy + y^2 = 3$$

a. No maximum, no minimum.
b. Maximum 4, minimum 1.
c. Maximum 5, minimum 3.
d. Maximum 7, minimum 3.
e. Maximum 5, minimum 1.
f. Maximum 3, minimum 1.
g. Maximum 6, minimum 1.
h. Maximum 6, minimum 3.
i. Maximum 7, minimum 1.
21. Find the maximum of $2x + 4y$ with $x^2 + y^2 \leq 5$.

   a. There is none.
   b. 12
   c. 10
   d. 8
   e. 6
   f. 4
   g. 2
   h. 3
   i. 5
   j. 7

22. Find the candidate for the maximum value of $-x^2 - y^2$ if $x - 3y \leq -10$.

   a. −1.
   b. −2.
   c. −3.
   d. −4.
   e. −5.
   f. −6.
   g. −7.
   h. −8.
   i. −9.
   j. −10.