This exam has 15 questions; indicate your answers on your answer card.

1. For the curve given by \( x^3 + x^2 - 2y^2 + 10y - 14 = 0 \) find the slope of the line tangent to the curve at the point \((1, 2)\).

   a. \(-9/2\)
   b. \(-7/2\)
   c. \(-5/2\)
   d. \(-3/2\)
   e. \(-1/2\)
   f. \(1/2\)
   g. \(3/2\)
   h. \(5/2\)
   i. \(7/2\)
   j. \(9/2\)

2. Find the equation of the plane tangent to the surface \( x^3 - 3x^2y^2 \) at the point \((x, y, z) = (2, 1, -4)\).

   a. \(z + 24y = 20\)
   b. \(6z + x - y = 30\)
   c. \(z + 3x + 4y = 15\)
   d. \(4z - x - 4y = 3\)
   e. \(z + 2x - 3y = 12\)
   f. \(2z - x = -6\)
   g. \(3z - 3x + 2y = -13\)
   h. \(3z + 3x - y = -3\)
   i. \(12z - 8x + y = 8\)
   j. \(4z - 5x + 6y = 8\)
3. There is a point in the region \(K, L > 0\) that maximizes the function 
\[20K^{1/4}L^{1/2} - 2L - K.\]
The coordinates of that point are \((K, L) = ?\)

a. \((125, 3025)\)
b. \((625, 625)\)
c. \((3025, 125)\)
d. \((15125, 25)\)
e. \((2, 1024)\)
f. \((4, 512)\)
g. \((8, 256)\)
h. \((16, 128)\)
i. \((32, 64)\)
j. \((64, 32)\)

4. Find and the maximum and minimum values of the function 
\[f(x, y) = x^4 + y^4\]
subject to the constraint \(x^2 + y^2 = 1.\)

a. There is no maximum, no minimum
b. There is no maximum, minimum = 2
c. There is no maximum, minimum = 1/2
d. There is no maximum, minimum = 1

e. Maximum = 2, there is no minimum
f. Maximum = 1, there is no minimum

g. Maximum = 1/2, there is no minimum
h. Maximum = 2, minimum = 1
i. Maximum = 2, minimum = 1/2
j. Maximum = 1, minimum = 1/2
5. Find and classify all the stationary points of the function

\[ f(x, y) = x^2 - y^3 + y^2 + 6 \]

a. Local maximum at \((0,0)\)
b. Local maximum at \((0,0)\)
c. Saddle point at \((0,0)\)
d. Local minimum at \((0,0)\), local maximum at \((0,2)\)
e. Local maximum at \((0,0)\), saddle point at \((0,3)\)
f. Local minimum at \((0,0)\), local minimum at \((0,3/2)\)
g. Saddle point at \((0,0)\), saddle point at \((0,3/2)\)
h. Saddle point at \((0,0)\), local maximum at \((0,2/3)\)
i. Local minimum at \((0,0)\), saddle point at \((0,2/3)\)
j. Local maximum at \((0,0)\), local maximum at \((0,2/3)\)

6. Find the coordinates of the point \((x,y)\) that gives the maximum value of \(\sqrt{2x} + \sqrt{y}\) subject to the condition \(x + y = 1\).

a. \((1,0)\)
b. \((3/4, 1/4)\)
c. \((2/3, 1/3)\)
d. \((1/2, 1/2)\)
e. \((1/3, 2/3)\)
f. \((2/5, 3/5)\)
g. \((1/4, 3/4)\)
h. \((1/6, 5/6)\)
i. \((5/6, 1/6)\)
j. \((0, 1)\)
7. Find the maximum and minimum values of the function
\[ f(x,y) = 12e^{-(4x^4+y^2)} \]

- a. There is no maximum, no minimum
- b. The maximum is 12, the minimum is 0.
- c. There is no maximum, the minimum is 0
- d. The maximum is 12, there is no minimum
- e. The maximum is $12/e$, there is no minimum.
- f. The maximum is $12e$, the minimum is $12/e$.
- g. The maximum is 0, there is no minimum.
- h. The maximum is $e$, the minimum is $-e$.
- i. The maximum is 0, the minimum is $-e$.
- j. There is no maximum, the minimum is $-e$.

8. Find the linear approximation (= linearization) of the function
\[ f(x,y) = \sqrt{4 - 2x^2 + 6y} \]
about the point (= at the point) (0,0).

- a. $2 + x - y$
- b. $2 + \frac{3}{4}y$
- c. $2 - x + \frac{3}{4}y$
- d. $2 - \frac{3}{4}y$
- e. $2 + x - 2y$
- f. $2 - x + \frac{3}{2}y$
- g. $2 + x - \frac{3}{2}y$
- h. $2 - \frac{3}{2}x$
- i. $2 - 2x + y$
- j. $2$
9. Solve the linear programming problem

maximize \( z = x + y \) subject to \( \begin{cases} \frac{x}{2} + \frac{y}{3} \leq 1, \\ \frac{x}{4} + y \leq 1 \end{cases} \) \( x, y \geq 0 \)

The maximum value of \( z \) is

a. \( \frac{7}{3} \)
b. \( \frac{9}{5} \)
c. \( \frac{9}{7} \)
d. \( \frac{11}{5} \)
e. \( \frac{8}{5} \)
f. \( \frac{5}{3} \)
g. \( \frac{7}{5} \)
h. \( \frac{12}{5} \)
i. \( \frac{14}{3} \)
j. \( \frac{9}{2} \)

10. Minimize \( 2x^2 + y^2 + z^2 \) subject to the condition \( x + y + z = 1 \).

a. \( \frac{8}{5} \)
b. \( \frac{7}{5} \)
c. \( \frac{5}{5} \)
d. \( \frac{2}{5} \)
e. \( \frac{1}{5} \)
f. \( \frac{1}{4} \)
g. \( \frac{3}{4} \)
h. \( \frac{3}{16} \)
i. \( \frac{7}{16} \)
j. \( \frac{9}{16} \)
11. Suppose that \( f(x,y) \) and \( g(x,y) \) are both homogenous functions of degree 5. What are the most general conditions on the numbers \( a, b \) which insure that the function

\[
h(x,y) = af(x,y) + bg(x,y)
\]

is homogenous of degree 5?

a. \( a = b = 0 \)
b. \( a = b = 5 \)
c. \( a = 0 \) or \( b = 0 \)
d. \( a + b = 0 \)
e. \( a - b = 0 \)
f. \( a + b = 5 \)
g. \( a - b = 5 \)
h. \( ab = 5 \)
i. \( a/b = 5 \)
j. Any values of \( a, b \).

12. Find the maximum and minimum of the function

\[
f(x,y) = 6xy + 8x - 14y + 5
\]

a. Minimum of 0, there is no maximum.
b. Minimum of 2, there is no maximum.
c. Minimum of 3, there is no maximum.
d. Minimum of 5, there is no maximum.
e. Minimum of 5, maximum of 10.
f. Minimum of 0, maximum of 10.
g. There is no minimum, maximum of 10.
h. There is no minimum, maximum of 20.
i. There is no minimum, maximum of 24.
j. There is no minimum, there is no maximum.
13. Suppose
\[ f(t, s) = e^{y^2} \] where \( y = t^2 s^3 \) and \( x = 2t + 3s \).

Find
\[ \frac{\partial}{\partial t} f(t, s) \]

at the point \((t, s) = (1, 0)\).

\[
\begin{align*}
\text{a.} & \quad 0 \\
\text{b.} & \quad 1 \\
\text{c.} & \quad 2 \\
\text{d.} & \quad 3 \\
\text{e.} & \quad 4 \\
\text{f.} & \quad 6 \\
\text{g.} & \quad 8 \\
\text{h.} & \quad 9 \\
\text{i.} & \quad 10 \\
\text{j.} & \quad 12
\end{align*}
\]

The next two questions concern the linear programing problem
\[
\text{maximize } z = 3x + 7y \text{ subject to the constraints } \begin{cases} 4x + y \leq 5, \\ x + 4y \leq 5 \\ x + 2y \leq 4 \end{cases} \text{ and with } x, y \geq 0.
\]

If you want to solve the problem it is enough to consider the values \( z \) in a certain region, the feasible (or admissible) region. That region is bounded by parts of the lines shown in the figure LINES on the next page. Question 14 on the next page asks which of the shaded regions in the pictures below LINES is the admissible region.

In fact, to solve the linear programming problem it is enough to consider the value of \( z \) at some of the points where the various lines in the figure LINES intersect. Question 15, two pages ahead, gives a labeling of those intersection points and asks exactly which intersection points need to be considered.
14. Which is the feasible region?
15. At which of the labeled points must the objective function \( z \) be evaluated in solving the given linear programming problem?

- a. 1, 3, 6, 10
- b. 7, 2, 3, 8
- c. 7, 4, 5, 8*
- d. 7, 1, 3, 6, 10
- e. 2, 3, 4, 5
- f. 1, 2, 3
- g. 3, 5, 6, 9, 10
- h. 1, 2, 3, 4, 5, 7, 8
- i. 3, 5, 8, 6, 9
- j. all ten points.