Final Exam, Math 128 - Dec 18, 2006

Name:
ID:
Discussion Section:

This exam has 12 multiple choice questions, 3 points each.

Important:

- No graphing calculators!
- You are allowed a 3×5 note card and the normal table for the exam.
- Please be careful with your calculations—each problem is valuable.

1. Find the point where $f(x, y) = 2x^2 + y^3 - x - 12y + 7$ has a relative minimum.

   (a) $(\frac{1}{4}, -2)$

   (b) $(\frac{1}{4}, 2)$ $\longrightarrow$ **CORRECT**

   (c) 2 or -2

   (d) $\frac{1}{4}$

$0 = \frac{\partial f}{\partial x} = 4x - 1$, $0 = \frac{\partial f}{\partial y} = 3y^2 - 12$ } critical points $(\frac{1}{4}, 2)$ $(\frac{1}{4}, -2)$

$\frac{\partial^2 f}{\partial x^2} = 4 > 0$ $\quad \frac{\partial^2 f}{\partial x \partial y} = 0$ $\quad \frac{\partial^2 f}{\partial y^2} = 6y$

$\Rightarrow D(\frac{1}{4}, 2) = 4 \cdot 12 - 0 > 0 \Rightarrow$ rel. min.

$D(\frac{1}{4}, -2) = 4 \cdot (-12) - 0 < 0 \Rightarrow$ neither min nor max.

2. Let $f(x, y) = (x + y^2)^3$. Calculate $\frac{\partial^2 f}{\partial x \partial y}(1, 2)$.

   (a) 300

   (b) 0

   (c) 36

   (d) 120 $\longrightarrow$ **CORRECT**

$\frac{\partial f}{\partial x} = 3(x+y^2)^2 \cdot 1$

$\frac{\partial^2 f}{\partial x \partial y} = 6(x+y^2) \cdot 2y$

$\Rightarrow \frac{\partial^2 f}{\partial x \partial y}(1,2) = 6 \cdot (1 + 4) \cdot 4 = 30 \cdot 4 = 120$

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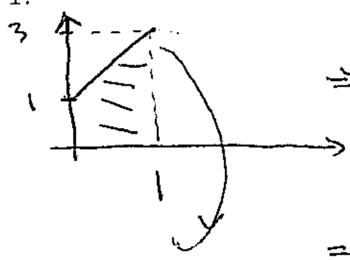
3. Find the volume of the solid of revolution generated by revolving about the x-axis the region under $y = 2x + 1$ from $x = 0$ to $x = 1$.

   (a) $\frac{13}{3}$

   (b) 2

   (c) $\frac{13\pi}{3}$ $\longrightarrow$ **CORRECT**

   (d) 2π



$$\Rightarrow V = \int_0^1 \pi (2x+1)^2 \, dx =$$

$$= \pi \int_0^1 (4x^2 + 4x + 1) \, dx =$$

$$= \pi \left(\frac{4}{3}x^3 + 2x^2 + x \right)\Big|_0^1 = \pi \left(\frac{4}{3} + 2 + 1 \right) =$$

$$= \frac{13}{3}\pi$$

4. Solve the initial value problem $t^2y' + ty = 2$, $y(1) = 1$, $t > 0$.

   (a) $y = \frac{t}{2\ln t + 1}$

   (b) $y = 2\ln t + C$

   (c) $y = \frac{2\ln t + 1}{t}$ $\longrightarrow$ **CORRECT**

   (d) $y = 2\ln t + 1$

$$u(t) = t^2 \cdot y(t)$$

$$\Rightarrow \frac{du}{dt} = t^2 y' + 2t \cdot y \Rightarrow$$ we need to use formula

$$t > 0 \Rightarrow y' + \frac{1}{t} y = \frac{2}{t^2}, \text{ i.e., } a(t) = \frac{1}{t}, \; b(t) = \frac{2}{t^2}$$

$$\Rightarrow A(t) = \ln t, \; e^{A(t)} = t, \; e^{-A(t)} = \frac{1}{t}$$

$$\Rightarrow y(t) = \frac{1}{t}\left[\int \cancel{t} \cdot \frac{2}{t^{\cancel{2}}} \, dt + c \right] = \frac{1}{t}\left[2\ln t + c \right]$$

$$1 = y(1) = \frac{1}{1}\left[2\underbrace{\ln 1}_{=0} + c \right] = c \Rightarrow y(t) = \frac{1}{t}\left[2\ln t + 1 \right]$$

5. Find

$$\int \frac{2-x}{\sqrt{2x^2-8x+1}}\,dx\ .$$

(a) $-\frac{1}{2}\sqrt{2x^2-8x+1}+C \longrightarrow$ **CORRECT**

(b) $-\frac{2}{3}\left(2x^2-8x+1\right)^{\frac{-3}{2}}+C$

(c) $-\frac{1}{2}\ln(2x^2-8x+1)+C$

(d) $-\frac{3}{2}\frac{1}{\sqrt{2x^2-8x+1}}+C$

substitution $u \quad u = 2x^2-8x+1$

$du = (4x-8)\,dx = (-4)(2-x)\,dx$

$\Rightarrow I = -\frac{1}{4}\int \frac{du}{\sqrt{u}} = -\frac{1}{4}\,2\sqrt{u} + C =$

$= -\frac{1}{2}\sqrt{2x^2-8x+1} + C$

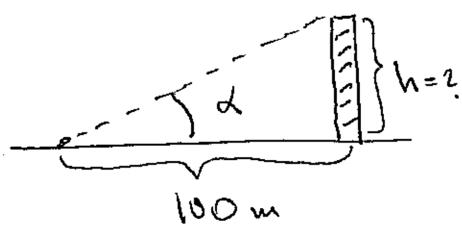
6. The angle of elevation from an observer to the top of a building is 29 degrees. If the observer is 100 meters from the base of the building, how high is the building (in meters)?

(a) 54.63 $\longrightarrow$ **CORRECT**

(b) 29

(c) $\frac{121}{2}$

(d) 11.73



$\alpha = 29°$

$\Rightarrow tg\,\alpha = \frac{h}{100}$

$29° \simeq .5$ radian

$\Rightarrow h = 100 \cdot tg(29°) = 54.63$

in meters

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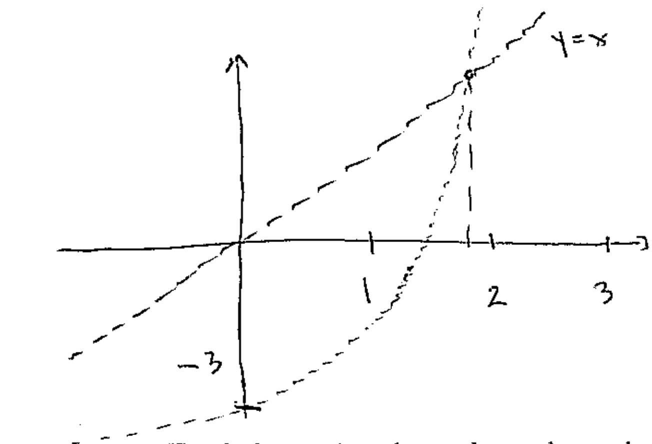
7. Approximate the positive solution of $e^x - 4 = x$. Let $x_0 = 2$ and find the third approximation (i.e., x_3) using the Newton - Raphson algorithm.

   (a) 1.78

   (b) 1.749 $\longrightarrow$ **CORRECT**

   (c) 1.21

   (d) 1.104



$x_0 = 2 \qquad f(x) = e^x - x - 4$

$f'(x) = e^x - 1$

$$x_1 = 2 - \frac{e^2 - 2 - 4}{e^2 - 1} \simeq 1.78$$

$x_2 \approx 1.75$

$x_3 \simeq 1.749$

8. Find the rational number whose decimal expansion is 4.011011.....

   (a) $\frac{11}{999}$

   (b) $\frac{4011}{1000}$

   (c) $\frac{4016}{1001}$

   (d) $\frac{4007}{999}$ $\longrightarrow$ **CORRECT**

$4.011011\ldots = 4 + .011011\ldots =$

$$= 4 + \frac{11}{1000} + \frac{11}{(1000)^2} + \cdots =$$

$$= 4 + \sum_{n=0}^{\infty} ar^n \quad \text{with } a = \frac{11}{1000}, \quad r = \frac{1}{1000}$$

$$\Rightarrow 4.011011\ldots = 4 + \frac{a}{1-r} =$$

$$= 4 + \frac{\frac{11}{1000}}{1 - \frac{1}{1000}} = 4 + \frac{11}{999} = \frac{4 \cdot 999 + 11}{999} = \frac{4007}{999}$$

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9. Find

$$\int x^2 \sin x \, dx \,.$$

(a) $-x^2 \cos x + 2x \sin x + C$

(b) $(2 - x^2) \cos x + 2x \sin x + C \longrightarrow$ **CORRECT**

(c) $\sin x - x \cos x + C$

(d) $-x^2 \cos x + C$

int. by parts

$f(x) = x^2 \Rightarrow f'(x) = 2x$

$g(x) = -\cos x \Rightarrow g'(x) = \sin x$

$\Rightarrow I = -x^2 \cos x + \int 2x \cos x \, dx =$ int. by parts again $=$

$= -x^2 \cos x + 2x \sin x - \int 2 \sin x \, dx =$

$= -x^2 \cos x + 2x \sin x + 2 \cos x + C =$

$= (2 - x^2) \cos x + 2x \sin x + C$

10. The number X of calls received by a telephone switchboard during a 1-minute interval is Poisson distributed with $\lambda = 5$. Determine the probability that three or more calls arrive during a particular minute.

(a) .87535 $\longrightarrow$ **CORRECT**

(b) .12465

(c) $\dfrac{1}{125}$

(d) .14062

We are looking for $P(X \geq 3)$ where X is Poisson with $\lambda = 5$.

$\Rightarrow P(X \geq 3) = 1 - (p_0 + p_1 + p_2) = 1 - .12465 = .87535$

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11. Find the expected value of the random variable whose probability density function is $f(x) = 12x(1-x)^2$, $0 \le x \le 1$.

(a) $\frac{4}{5}$

(b) $\frac{1}{2}$

(c) $\frac{2}{5}$ $\longrightarrow$ CORRECT

(d) 1

$$EX = \int_0^1 x \cdot f(x)\,dx = \int_0^1 12x^2(1-x)^2\,dx =$$

$$= \int_0^1 (12x^4 - 24x^3 + 12x^2)\,dx =$$

$$= \left(\frac{12}{5}x^5 - \frac{24}{4}x^4 + \frac{12}{3}x^3\right)\Big|_0^1 =$$

$$= \frac{12}{5} - 6 + 4 = \frac{2}{5}.$$

12. The men hired by a certain city police department must be at least 69 inches tall. Suppose that the heights of adult men in the city are normally distributed with $\mu = 70$ inches and $\sigma = 2$ inches. What percentage of the men are tall enough to be eligible for recruitment by the police department?

(a) 80.85%

(b) 70.88%

(c) 79.12%

(d) 69.15% $\longrightarrow$ CORRECT

Consider $Y = \frac{X-\mu}{\sigma} \Rightarrow Y$ standard normal

$$P(X \geq 69) = P\left(Y \geq \frac{69-70}{2}\right) =$$

$$= P(Y \geq -.5) = .5 + A(.5) = \text{(Table 1)}$$

$$= .5 + .1915 = .6915.$$