This exam contains fourteen questions. The first twelve are multiple choice questions and count for five points each. There is no partial credit on these questions, so read each question carefully, check your arithmetic and make sure that you have marked the answer you intended to mark. The last two questions, which are each worth twenty points, require written answers, and some partial credit might be given. However, no credit will be given for information that is not germane to the problem at hand. Please make sure to write your name and student ID number on the pages that include your answers to the last two questions. In fact, you will get one point on each of these two questions for writing your name and ID number legibly.
1. What is \( \lim_{x \to 2} 3x + 1 \)?

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5
(f) 6
(g) 7
(h) the limit does not exist.
2. What is the natural domain for \( f(x) = \sqrt{2x - 1} \)?

(a) \([2, \infty)\)  
(b) \((2, \infty)\)  
(c) \((0, \infty)\)  
(d) \([0, \infty)\)  
(e) \(\left(\frac{1}{2}, \infty\right)\)  
(f) \(\left[\frac{1}{2}, \infty\right)\)  
(g) \((-\infty, 0)\)  
(h) \((-\infty, 0]\)
3. Say \( f(x) = \frac{1}{x} \) and \( g(x) = x^2 - 3 \). Which of the following is true?

(a) \( f \circ g(x) = \frac{1}{x^2 - 3} \) and \( g \circ f(x) = \frac{1}{x} \).
(b) \( f \circ g(x) = \frac{1}{x^2 - 3} \) and \( g \circ f(x) = \frac{1}{x^2} - 3 \).
(c) \( f \circ g(x) = x^2 - 3 \) and \( g \circ f(x) = \frac{1}{x} \).
(d) \( f \circ g(x) = \frac{1}{x^2} - 3 \) and \( g \circ f(x) = \frac{1}{x^2 - 3} \).
(e) \( f \circ g(x) = g \circ f(x) = \frac{1}{x^2 - 3} \).
(f) \( f \circ g(x) = g \circ f(x) = \frac{1}{x^2} - 3 \).
(g) \( f \circ g(x) = x \) and \( g \circ f(x) = (x^2 - 3)^2 - 3 \).
(h) \( f \circ g(x) = g \circ f(x) = \frac{1}{x^2} + x^2 - 3 \).
4. Say \( f(x) = \frac{1}{x} \) and \( g(x) = \frac{x-2}{x^2-1} \). Which are the only real numbers *not* in the natural domain of \( f \circ g \)?

(a) 0, 1, −1.
(b) 1, −1.
(c) 1, 2, −1.
(d) 0.
(e) 0, 2.
(f) 2.
(g) 0, 1, 2, −1.
(h) All real numbers are in the domain.
5. What is 

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}?$$

(a) $-2$
(b) $-1$
(c) 0
(d) 1
(e) 2
(f) 3
(g) 4
(h) The limit does not exist.
6. Say \( f(x) = x^2 \). What is

\[
\lim_{h \to 0} \frac{f(2 + h) - f(2)}{h}
\]

(a) 4
(b) \( 2x \)
(c) \( 2h \)
(d) 2
(e) 1
(f) \( x^2 \)
(g) \( h^2 \)
(h) The limit does not exist.
7. Which of the following functions has a limit as \( x \) approaches 2?

\[
f(x) = \begin{cases} 
-1 & x < 2 \\
1 & x \geq 2 
\end{cases}
\]

\[
g(x) = \begin{cases} 
\frac{x^2-4}{x-2} & x \neq 2 \\
5 & x = 2 
\end{cases}
\]

\[
h(x) = \frac{x^2 - 4}{x + 2}
\]

(a) All of \( f, g, h \) have a limit.
(b) \( f \) and \( g \) have a limit, but \( h \) does not.
(c) \( f \) and \( h \) have a limit, but \( g \) does not.
(d) \( g \) and \( h \) have a limit, but \( f \) does not.
(e) \( f \) has a limit, but \( g \) and \( h \) do not.
(f) \( g \) has a limit, but \( f \) and \( h \) do not.
(g) \( h \) has a limit, but \( f \) and \( g \) do not.
(h) None of \( f, g, h \) has a limit.
8. Which of the following functions is continuous at 2?

\[ f(x) = \begin{cases} 
-1 & x < 2 \\
1 & x \geq 2 
\end{cases} \]

\[ g(x) = \begin{cases} 
\frac{x^2 - 4}{x^2} & x \neq 2 \\
5 & x = 2 
\end{cases} \]

\[ h(x) = \frac{x^2 - 4}{x + 2} \]

(a) All of \( f, g, h \) are continuous at 2.
(b) \( f \) and \( g \) are continuous at 2, but \( h \) is not.
(c) \( f \) and \( h \) are continuous at 2, but \( g \) is not.
(d) \( g \) and \( h \) are continuous at 2, but \( f \) is not.
(e) \( f \) is continuous at 2, but \( g \) and \( h \) are not.
(f) \( g \) is continuous at 2, but \( f \) and \( h \) are not.
(g) \( h \) is continuous at 2, but \( f \) and \( g \) are not.
(h) None of \( f, g, h \) is continuous at 2.
9. What are all real numbers $c$ such that the function

$$f(x) = \begin{cases} \frac{1}{x+c} & x < 0 \\ c(x+1) & x \geq 0 \end{cases}$$

is continuous at 0?

(a) $1, -1$
(b) $1$
(c) $-1$
(d) $0, 1, -1$
(e) $0, 1$
(f) $0, -1$
(g) $0$
(h) There are no such numbers.
10. What is
\[
\lim_{x \to \infty} \frac{2x^2 - 5}{3x^2 + 10000^2}
\]
(a) 1
(b) \(\frac{2}{3}\)
(c) \(\frac{3}{2}\)
(d) \(-\frac{5}{10000}\)
(e) \(-\frac{3}{10003}\)
(f) \(\frac{7}{10003}\)
(g) \(\frac{2}{10003}\)
(h) the limit does not exist.
11. Say $f$ is a function whose domain is the set of all real numbers. If $g(x) = f(x + 3)$ then the graph of $g$ is obtained from that of $f$ by

(a) shifting three units up.
(b) shifting three units down.
(c) shifting three units to the left.
(d) shifting three units to the right.
(e) stretching along the $y$ axis by a factor of three.
(f) shrinking along the $y$ axis by a factor of three.
(g) stretching along the $x$ axis by a factor of three.
(h) shrinking along the $x$ axis by a factor of three.
12. Fill in all four blanks in the following paragraph:

The slope of the tangent line to the graph of \( f(x) = x^2 \) at the point \((1, 1)\) is the limit as \( x \) approaches (FIRST BLANK) of the function (SECOND BLANK). The value of this limit is (THIRD BLANK), so the equation of the tangent line is (FOURTH BLANK).

(a) \( 0, x^2, 0, y = 1 \)
(b) \( 1, \frac{(x-1)^2-1}{x}, 2, y = 2x \)
(c) \( 1, \frac{x^2-1}{x-1}, 2, y = 2x - 1 \)
(d) \( 1, \frac{x^2}{x}, 1, y = x \)
(e) \( 0, \frac{x^2}{x}, 0, y = 0 \)

(f) I don’t know the answer because I wasn’t in class when Professor Shareshian pretty much said that there would be a problem like this on the test.

(g) I don’t know the answer because I got my friend who already took this class to do the hard webwork problems for me.

(h) I don’t know the answer, but I would like to inject at this juncture that Professor Shareshian is an absolutely outstanding teacher whose salary should be doubled immediately.
13. State the precise definition of a limit (using $\epsilon$ and $\delta$) and use the definition to prove that

$$\lim_{x \to 1} (3x - 1) = 2.$$
14. State precisely the definition of continuity of a function $f$ at a point $c$. Then find all points at which the following function is not continuous, and explain why $f$ is not continuous at these points using the definition. (Note - if you calculate some limits in order to answer this question, you do not need to prove that your calculations are correct using the $\epsilon, \delta$ definition of a limit.)

$$f(x) = \begin{cases} 
-1 & x < 0 \\
x & 0 \leq x < 1 \\
2 & x = 1 \\
x & 1 < x \leq 2 \\
2(x - 1) & x > 2 
\end{cases}$$