This exam should have 16 questions. Part I will have 12 multiple choice questions, 5 points each. Part II will have 4 hand graded questions, 10 points each. Please check to see that your exam is complete. If you do not have a PENCIL to mark your card, please ask to borrow one from your proctor.

Write your ID number (not your SS number) on the six blank lines at the top of your answer card, using one blank for each digit. Shade in the corresponding boxes below. Also print your name at the top of your card. If your card becomes damaged please ask your proctor for a new one.

You may use a scientific calculator (not one with graphing). You're also allowed to bring with you a 3x5 card with any data that you can fit on it.

PART I: (60 points)

1. Given \( \ln(y+2) - \ln(4) = \ln(x^2) + \ln(2) \)

   solve for \( y \) in terms of \( x \).

A) \( y = 4e^{2x^2} - 2 \)

B) \( y = 2e^{2x^2} + 4 \)

C) \( y = 4e^{x^2+2} - 2 \)

D) \( y = 2e^{x^2+2} + 4 \)

E) \( y = 4x^2 + 2 \)

F) \( y = 8x^2 - 2 \)

G) \( y = x^2 - 8 \)

H) \( y = \frac{1}{8}x^2 + 4 \)

I) \( y = e^{x^2} - 2 \)

J) \( y = e^{2x^3} - 8 \)

\[
\ln\left(\frac{y+2}{4}\right) = \ln\left(2x^2\right)
\]

\[
\frac{y+2}{4} = 2x^2
\]

\[
y + 2 = 8x^2
\]

\[
y = 8x^2 - 2
\]
2. Find \( \lim_{h \to 0} \frac{(1 + \frac{1}{x})^2 - \frac{1}{x}}{h} \). (Hint: It is easier to do if you notice that it's the derivative of a function \( f(x) \) at some number, \( a \). Find \( f(x) \) and \( a \) and use a derivative formula.

A) \( \frac{1}{4} \)

B) \( -\frac{1}{4} \)

C) \( \frac{1}{8} \)

D) \( -\frac{1}{8} \)

E) \( \frac{1}{16} \)

F) \( -\frac{1}{16} \)

G) \( \frac{1}{32} \)

H) \( -\frac{1}{32} \)

Easier way: \( \lim_{h \to 0} \frac{\left(\frac{1}{x+h}\right)^2 - \frac{1}{x}}{h} \) is

\( f'(x) = \frac{1}{x^2} \),

\( f'(x) = -2x^{-3} = -\frac{2}{x^3} \)

\( f'(4) = -\frac{2}{64} = -\frac{1}{32} \)
3.

Given \( f(x) = \begin{cases} 
2x^2 - x & \text{if } x \leq 2 \\
2x & \text{if } x > 2
\end{cases} \)

Find \( \lim_{x \to 2^+} f(x) \), if it exists.

A) 1
B) 2
C) 3
D) 4
E) 5
F) 6
G) 7
H) 8
I) 9
J) DNE

\[ \lim_{x \to 2^+} f(x) = 4 \] (even though \( f(2) = 6 \))
4. Find the slope of the secant line of the curve \( y = x^3 - 4x^2 + 5x \) when \( a = 1 \) and \( h = 0.1 \)

\[
\frac{f(a+h) - f(a)}{h} = \frac{f(1.1) - f(1)}{0.1} = \frac{1.991 - 2}{0.1} = -0.09
\]
5. Find \( \lim_{x \to -3} \frac{x^2 + x - 6}{x + 3} \).

A) 0
B) 1
C) -1
D) 2
E) -2
F) 4
G) -4
H) 5
I) -5

\[
\begin{align*}
\lim_{x \to -3} \frac{x^2 + x - 6}{x + 3} &= \lim_{x \to -3} \frac{(x+3)(x-2)}{(x+3)} \\
&= \lim_{x \to -3} (x-2) \\
&= -5
\end{align*}
\]
6. List all the horizontal and vertical asymptotes of the curve \( y = \frac{x^3 - 3x + 2}{x^2 + 2x} \).

A) \( y = 1, \ x = 1, \ x = 2 \)
B) \( y = 1, \ x = 0, \ x = -2 \)
C) \( y = 2, \ x = -1, \ x = 1 \)
D) \( y = 0, \ x = -1, \ x = 0 \)
E) \( y = 1, \ x = -1, \ x = 1 \)
F) \( y = 2, \ x = 0, \ x = 2 \)
G) \( y = 1, \ x = 2 \)
H) \( y = 1, \ x = -2 \)
I) \( y = 2, \ x = 0 \)
J) \( y = 1, \ x = 0 \)

**Horizontal:**

\[
\lim_{x \to \pm\infty} \frac{x^2 - 3x + 2}{x^2 + 2x} = \lim_{x \to \pm\infty} \frac{1 - \frac{3}{x} + \frac{2}{x^2}}{1 + \frac{2}{x}}
\]

So \( y = 1 \) is only horizontal asymptote.

**Vertical:** \( x^2 + 2x = x(x + 2) = 0 \)

for \( x = 0 \) and \( x = -2 \)

(numerator is not zero at \( x = 0 \) or \( x = -2 \))

\( x = 0 \) and \( x = -2 \) are the two vertical asymptotes.
7. Find \( \lim_{x \to -2} \frac{x^2 + 2}{\sqrt{x^2 + 5} - 3} \), if it exists.

A) 0

B) \( \frac{1}{2} \)

C) \( -\frac{1}{4} \)

D) \( \frac{2}{3} \)

E) \( -\frac{3}{2} \)

F) \( \frac{4}{5} \)

G) \( -\frac{5}{4} \)

H) 1

I) -1

J) DNE

\[
= \lim_{x \to -2} \frac{x^2 + 2}{\sqrt{x^2 + 5} - 3} \cdot \frac{(\sqrt{x^2 + 5} + 3)}{(\sqrt{x^2 + 5} + 3)}
\]

\[
= \lim_{x \to -2} \frac{(x + 2)(\sqrt{x^2 + 5} + 3)}{x^2 + 5 - 9}
\]

\[
= \lim_{x \to -2} \frac{(x + 2)(\sqrt{x^2 + 5} + 3)}{x^2 - 4}
\]

\[
= \lim_{x \to -2} \frac{(x + 2)(\sqrt{x^2 + 5} + 3)}{(x + 2)(x - 2)}
\]

\[
= \lim_{x \to -2} \frac{\sqrt{x^2 + 5} + 3}{x - 2}
\]

\[
= \frac{6}{-4}
\]

\[
= -\frac{3}{2}
\]
8. Find the $x$-coordinates of all the points on the curve $y = x^3 - 6x^2 + 9x$ where the tangent line is horizontal.

A) $x = 0, \ x = 1$
B) $x = 1, \ x = 2$
C) $x = 2, \ x = 3$
D) $x = -1, \ x = 1$
E) $x = -2, \ x = 1$
F) $x = -1, \ x = 3$

**G) $x = 1, \ x = 3$**
H) $x = 0, \ x = 3$
I) $x = 2, \ x = 4$

Horizontal tangent line means the slope, $f'(x)$, is zero.

$f'(x) = 3x^2 - 12x + 9 = 0$

$3(x^2 - 4x + 3) = 0$

$3(x-1)(x-3) = 0$

$x=1$ and $x=3$
9. Find \( \lim_{x \to \infty} \left( \sqrt{9x^2 + x} - 3x \right) \) if it exists.

A) 0  
B) 3  
C) \( \frac{1}{3} \)  
D) 6  
E) \( \frac{1}{6} \)  
F) 8  
G) \( \frac{1}{8} \)  
H) 9  
I) \( \frac{1}{9} \)  
J) DNE

\[
\lim_{x \to \infty} \frac{9x^2 + x - 9x^2}{\sqrt{9x^2 + x} + 3x} = \lim_{x \to \infty} \frac{X}{X \sqrt{9 + \frac{1}{X}} + 3X} = \frac{1}{6}
\]
10. If \( f(x) = (x^{5/2} + \frac{1}{x})(2x^2 - x^{3/2}) \) then \( f'(1) = \):

A) \( \frac{1}{2} \)

B) \( \frac{3}{2} \)

C) \( \frac{5}{2} \)

D) \( \frac{7}{2} \) "product rule"

E) \( \frac{9}{2} \)

F) \( \frac{11}{2} \)

G) \( \frac{13}{2} \)

H) \( \frac{15}{2} \)

I) \( \frac{17}{2} \)

J) \( \frac{19}{2} \)

\[
f'(x) = \left( \frac{5}{2} x^{3/2} - \frac{1}{x^2} \right) \left( 2x^2 - x^{3/2} \right) + \left( x^{5/2} + \frac{1}{x} \right) \left( 4x - \frac{3}{2} x^{1/2} \right)
\]

\[
f'(1) = \left( \frac{5}{2} - 1 \right) \left( 2 - 1 \right) + \left( 1 + 1 \right) \left( 4 - \frac{3}{2} \right) = \frac{3}{2} + 5 = \frac{13}{2}
\]
11. Suppose that $f(2) = 6$, $g(2) = 4$, $f'(2) = \frac{1}{2}$, $g'(2) = \frac{1}{4}$.

If both are differentiable at $x = 2$ then $(\frac{f}{g})'(2) =$: 

(i.e. compute the derivative of the quotient at $x = 2$.)

A) $\frac{1}{4}$  
B) $-\frac{1}{4}$  
C) $\frac{1}{8}$  
D) $-\frac{1}{8}$  
E) $\frac{1}{16}$  
F) $-\frac{1}{16}$  
G) $\frac{1}{24}$  
H) $-\frac{1}{24}$  
I) $\frac{1}{32}$  
J) $-\frac{1}{32}$

By quotient rule

\[
\left(\frac{f}{g}\right)'(2) = \frac{f'(2) \cdot g(2) - f(2) \cdot g'(2)}{g(2)^2}
\]

\[
= \frac{\left(\frac{1}{2}\right)(4) - (6)(\frac{1}{4})}{4^2}
\]

\[
= \frac{2 - \frac{3}{2}}{16} = \frac{1}{32}
\]
12. Suppose \( f(x) = \begin{cases} 
  x^2 - c & \text{if } x \leq 2 \\
  e^{x - 5} & \text{if } x > 2 
\end{cases} \) for some constant \( c \).

Find the constant \( c \) given that \( \lim_{x \to 2} f(x) \) exists.

If \( \lim_{x \to 2} f(x) \) exists, it means that

\[
\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)
\]

\[
\text{i) } \lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} e^{x - 5} = 2c - 5
\]

\[
\text{ii) } \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 - c = 4 - c
\]

So \( 2c - 5 = 4 - c \)

\[ 3c = 9 \]

\[ c = 3 \]
Part II: (40% of test points) In each problem, clearly show your solution in the space provided. Present a sequence of steps showing how you got the answer.

**PLEASE PUT YOUR NAME AND ID NUMBR ON EACH OF THE SHEETS OF PART II** (IN CASE THE SHEETS GET SEPERATED).

13. a) Given that \( \frac{\sqrt{16} + 0.01 - 2}{0.01} \) represents the slope of a secant line of some curve \( f(x) \). **What is \( f(x) \)?** (3 pts)

   \[
   \text{slope of secant line is } \frac{f(a+h) - f(a)}{h} \\
   \text{So } \quad f(x) = \sqrt{x} \\
   \text{also } \quad a = 16
   \]

b) **Which 2 points** on the curve does that secant line connect? (3 pts)

   \[
   (a+h, f(a+h)) \quad \text{and} \quad (a, f(a)) \quad \text{are} \\
   (16, 01, \sqrt{16.01}) \quad \text{and} \quad (16, 2) \\
   \text{OR} \quad (16, 01, 2.000312427) \quad \text{and} \quad (16, 2)
   \]

c) Find \( \lim_{h \to 0} \frac{\sqrt{16 + h} - 2}{h} \) (It's the slope of a tangent line.) (4 pts)

   \[
   f(x) = \sqrt{x} \quad f'(x) = \frac{1}{2} x^{-\frac{3}{2}} \\
   f'(16) = \frac{1}{4 \cdot 16^{\frac{3}{2}}} = \frac{1}{4 \cdot 8} = \frac{1}{32}
   \]
14. a) Use the formula \( f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \) to compute \( f'(-1) \) for the function \( f(x) = 6x^2 - 6x \).
(Must use a limit calculation, you may not use any derivative rule.) (5 pts)

\[
\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)}
\]

\[
= \lim_{x \to -1} \frac{6x^2 - 6x + 12}{x + 1}
\]

\[
= \lim_{x \to -1} \frac{6(x+1)(x-2)}{x+1}
\]

\[
= \lim_{x \to -1} \frac{6(x-2)}{x+1}
\]

\[
= -18
\]

b) Find the x-coordinates of all points on the graph of \( y = 2x^3 - 3x^2 \), where the tangent lines at those points are parallel to the line \( 12x - y = 8 \). (5 pts).

Slope of tangent line is \( \frac{dy}{dx} = 6x^2 - 6x \)

Slope of line \( 12x - y = 8 \) \( (y = 12x - 8) \) is 12

So we need to solve \( 6x^2 - 6x = 12 \)

\( 6x^2 - 6x - 12 = 0 \)

\( 6(x+1)(x-2) = 0 \) \( (\text{as above}) \)

\( x = -1 \) and \( x = 2 \)
PART II (cont.)

Name ______________________________________

ID number ______________________

15) a) A radioactive substance has a half-life of 40 years and we start with 240 kg of the substance. If \( A = f(t) \) is the amount remaining after \( t \) years, then compute \( f(40) \), \( f(80) \), \( f(120) \) and \( f(160) \). (4 pts)

\[
\begin{align*}
f(40) &= \frac{1}{2} (240) = 120 \\
f(80) &= \frac{1}{2} (120) = 60 \\
f(120) &= \frac{1}{2} (60) = 30 \\
f(160) &= \frac{1}{2} (30) = 15
\end{align*}
\]

b) Given that the general formula is \( A = f(t) = C \cdot e^{kt} \), find the values of \( C \) and \( k \). (6 pts)

Given \( C = f(0) = 240 \)

So \( f(t) = 240 \cdot e^{kt} \)

Given \( 120 = f(40) = 240 \cdot e^{40k} \)

\( \frac{1}{2} = e^{40k} \)

\( 40k = \ln\left(\frac{1}{2}\right) = -\ln(2) \)

\( k = -\frac{\ln(2)}{40} \)
16. Using the product and quotient rules, compute the following derivatives. Just compute the derivative, don't put it in a simpler form.

a) If $f(x) = (x^{2/3} - \frac{1}{x})(x^{11} - \sqrt{x})$, find $f'(x)$. (3 pts)

$$f'(x) = \left( x^{2/3} - \frac{1}{x} \right) \left( x^{11} - \frac{1}{\sqrt{x}} \right)$$

$$f'(x) = \left( \frac{2}{3} x^{-1/3} + x^{-2} \right) \left( x'' - \frac{1}{2} x^{-3/2} \right) + \left( \frac{11}{x} - x^{-1} \right) \left( \frac{1}{2} x^{-3/2} + \frac{1}{2} x^{-3/2} \right)$$

or

$$f'(x) = \left( \frac{2}{3} x^{-1/3} + \frac{1}{x^2} \right) \left( x'' - \frac{1}{2} x^{-1/2} \right) + \left( x^{-1} - \frac{1}{x} \right) \left( \frac{1}{2} x^{-3/2} + \frac{1}{2} x^{-3/2} \right)$$

b) If $y = \frac{t^3 - 2t}{t^2 - 4}$ then find $\frac{dy}{dt}$. (4 pts)

$$\frac{dy}{dt} = \frac{3t^2 - 4t}{(t^2 - 4)^2} \left( (t^4 - 4) - (t^2 - 2t)(4t^3) \right)$$

$$\left( t^4 - 4 \right)^2$$

c) If $f(x) = x e^x$ then find the second derivative $f''(x)$. (3 pts)

$$f'(x) = e^x + xe^x$$

$$f''(x) = e^x + e^x + xe^x$$

$$f''(x) = 2e^x + xe^x$$