PART I consists of 12 multiple choice questions (worth 5 points each) for a total of 60 points. Mark the correct answer on the answer card. For part I only the answer on the card will be graded.

Part II consists of 4 hand-graded problems (worth 10 points each) for a total of 40 points. A correct answer without supporting work may get no credit. Present a readable, orderly sequence of steps showing how you got your answer.

Part I (60 points):

1) If \( f(x) = \frac{1}{\sqrt{x^2 + x + 2}} \) then \( f'(2) = \):

A) 0  B) -2  C) \( \frac{3}{25} \)  D) -\( \frac{5}{18} \)  E) \( \frac{1}{4} \)  F) -\( \frac{3}{17} \)  G) \( \frac{3}{4} \)  H) -\( \frac{5}{48} \)  I) \( \frac{9}{52} \)  J) \( \frac{1}{2} \)

\[
f(x) = \left( x^2 + x + 2 \right)^{-\frac{1}{2}}
\]

\[
f'(x) = -\frac{1}{2} \left( x^2 + x + 2 \right)^{-\frac{3}{2}} \left( 2x + 1 \right)
\]

\[
f'(2) = -\frac{5}{3 \cdot 8^{1/3}} = -\frac{5}{48}
\]
2) Find the slope of the tangent line to the curve \( x^2 - 2xy + y^3 = 1 \) at \((2, 1)\).

\[
2x - 2y - 2xy' + 3y^2y' = 0
\]

\[
x = 2, \quad y = 1
\]

\[
y - 2 - 4y' + 3y = 0
\]

\[
2 = y'
\]

3) Find the slope of the tangent line to the curve with parametric equations

\[
x = t \sin(t), \quad y = t \cos(t)
\]

at the point \((0, -\pi)\), if \(\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}\).

\[
A) \pi \quad B) -\pi \quad C) 2\pi \quad D) -2\pi \quad E) \frac{\pi}{2} \quad F) -\frac{\pi}{2} \quad G) \frac{1}{4} \quad H) -\frac{1}{4} \quad I) \frac{1}{3\pi} \quad J) -\frac{1}{3\pi}
\]

\[
\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos(t) - t \sin(t)}{\sin(t) + t \cos(t)}
\]

\[
x = 0, \quad y = -\pi \quad \Rightarrow \quad t = \pi
\]

\[
\frac{dy}{dx} = \frac{-1}{-\pi} = \frac{1}{\pi}
\]
4) Eliminate the parameter to find a Cartesian Equation of the curve
\[ x = 2 \cos(t) \quad y = 3 \sin(t) \].

\[ \frac{x}{2} = \cos(t) \quad \frac{y}{3} = \sin(t) \]

\[ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = \cos^2(t) + \sin^2(t) = 1 \]

\[ \frac{x^2}{4} + \frac{y^2}{9} = 1 \]

\[ 9x^2 + 4y^2 = 36 \]

5) Find the angle given by \( \text{arcsec} \left( -\frac{2}{\sqrt{3}} \right) \) (Remember the range of \( \text{arcsec}(x) \)).

\[ A) 0 \quad B) \frac{\pi}{6} \quad C) -\frac{\pi}{6} \quad D) \frac{2\pi}{3} \quad E) -\frac{2\pi}{3} \quad F) \frac{5\pi}{6} \quad G) -\frac{5\pi}{6} \quad H) \frac{7\pi}{6} \quad I) -\frac{7\pi}{6} \quad J) \frac{3\pi}{4} \]

\[ \frac{\pi}{2} < \text{arcsec} \left( -\frac{2}{\sqrt{3}} \right) < \pi \]

\[ \text{If} \quad \theta = \text{arcsec} \left( -\frac{2}{\sqrt{3}} \right) \quad \text{Then} \]

\[ \sec(\theta) = -\frac{2}{\sqrt{3}} \quad \Rightarrow \quad \cos(\theta) = -\frac{\sqrt{3}}{2} \]

\[ \theta = \frac{5\pi}{6} \]
6) If \( f(x) = (\sin^{-1}(x))^2 \) then \( f'(\frac{1}{2}) = \):

\[ f'(x) = 2 \left( \sin^{-1}(x) \right) \cdot \frac{1}{\sqrt{1-x^2}} \]

\[ f'\left(\frac{1}{2}\right) = 2 \cdot \sin^{-1}\left(\frac{1}{2}\right) \cdot \frac{1}{\sqrt{1-\frac{1}{4}}} \]

\[ = 2 \cdot \frac{\pi}{6} \cdot \frac{1}{\sqrt{\frac{3}{4}}} \]

\[ = \frac{\pi}{3} \cdot \frac{2}{\sqrt{3}} = \]

A) 1.209
B) 1.376
C) 1.482
D) 1.594
E) 1.645
F) 1.763
G) 1.804
H) 1.946
I) 2.056
J) 2.874

7) If \( f(x) = \arctan(\sin(x)) \) then \( f'(\pi/3) = \):

\[ f'(x) = \frac{1}{1 + \sin^2(x)} \cdot \cos(f(x)) \]

\[ f'(\frac{\pi}{3}) = \frac{\frac{1}{2}}{1 + \frac{3}{4}} = \frac{\frac{1}{2}}{\frac{7}{4}} = \frac{2}{7} \]
8) Find the **slope** of the tangent line to the curve $4 \cos(x) \sin(y) = 2$ at the point $x = \frac{\pi}{4}, \ y = \frac{\pi}{4}$.

- $y' = 0$
- $-4 \sin(x) \sin(y) + 4 \cos(x) \cos(y) \cdot y' = 0$
- $x = \frac{\pi}{4}, \ y = \frac{\pi}{4}$
- $-4 \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + 4 \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} y' = 0$
- $y' = 0$
- $y = 1$

9) The position of a particle moving along the x-axis is given by the formula $x = t^3 - 6 t^2 + 9 t$ m, t in seconds. Find the particle's **accelerations** at the two times that the velocity is zero.

- $v = \frac{dx}{dt} = 3t^2 - 12t + 9 = 0$
- $3 (t^2 - 4t + 3) = 0$
- $3 (t-1)(t-3) = 0$
- $t = 1$ or $t = 3$

- $a = \frac{dv}{dt} = 6t - 12$
- $a = \begin{cases} -6 & t = 1 \\ 6 & t = 3 \end{cases}$
10) If \( f(x) = \ln(\ln(x)) \) then find \( f'(e) \).

\[
f'(x) = \frac{1}{\ln(x)} \cdot \frac{1}{x}
\]

\[
f'(e) = 1 \cdot \frac{1}{e}
\]

A) \( e \)
B) \( -e \)
C) \( \frac{1}{e} \)
D) \( -\frac{1}{e} \)
E) \( e^2 \)
F) \( -e^2 \)
G) 1
H) \(-1\)
I) \(2e\)
J) \(-2e\)

11) If \( F(x) = x^x \), find \( F'(2) \).

\[
y = x^x
\]

\[
\ln(y) = x \ln(x)
\]

\[
\frac{1}{y} \cdot y' = \ln(x) + 1
\]

\[
y' = y^x(\ln(x) + 1)
\]

\[
x = 2 \quad y' = 2^2 \left( \ln(2) + 1 \right)
\]

\[
\approx \frac{6.7726}{6.7726}
\]

A) 6.3429
B) 6.7726
C) 7.2465
D) 7.5879
E) 7.9898
F) 8.3490
G) 8.8198
H) 9.1245
I) 9.5439
J) 9.8765
12) Suppose that the length of a rectangle is decreasing at the rate of 3 cm/s while the width is increasing at the rate of 4 cm/s. Find the rate of change of the area when the length is 6 cm and the width is 4 cm.

\[ A = l \cdot w \]

\[ \frac{dA}{dt} = \frac{dl}{dt} \cdot w + l \cdot \frac{dw}{dt} \]

\[ = (-3)(4) + (6)(4) \]

\[ = 12 \text{ cm}^2/\text{s} \]
PART II: Show the work you did to get the answer in a readable orderly form.

13) A ball is thrown upward at time $t=0$ from the edge of the roof of a building, 192 ft above ground. After rising a bit it passes the roof on its way down to the ground. The formula for its height (in feet) above the ground at time $t$ (in sec.) is given by $s = f(t) = -16t^2 + 64t + 192$ (e.g. when $t=0$, $s = 192$).

a) Find formulas for velocity and acceleration at time $t$. (2 pts)

$$\begin{align*}
\text{Velocity: } v &= -32t + 64 \text{ ft/sec} \\
\text{Acceleration: } a &= -32 \text{ ft/sec}^2
\end{align*}$$

b) At what time does it reach the ground? (4 pts)

Solve for $s = 0$

$$-16t^2 + 64t + 192 = 0$$

$$-16(t^2 + 4t - 12) = 0$$

$$-16(t - 6)(t + 2) = 0$$

$$t = 6 \text{ Seconds}$$

c) What is its velocity as it passes the roof on its way down? (4 pts)

Find $v$ when $s = 192$

$$s = 192 = -16t^2 + 64t + 192$$

$$0 = -16t^2 + 64t$$

$$0 = -16t(t - 4)$$

$$t = 0 \text{ (when it started)}$$

$$t = 4 \text{ (on the way down)}$$

$$v = -32(4) + 64$$

$$v = -128 + 64 = \sqrt{-64} \text{ ft/s}$$

Solved is $64 \text{ ft/s}$
PART II: Show the work you did to get the answer in a readable orderly form.

14) The curve \( y^2 = x^3 + 3x^2 \) is called the Tschirnhausen cubic.

   a) Find an equation of the tangent line to this curve at the point \((1, -2)\). (5 pts)

\[
2y \frac{dy}{dx} = 3x^2 + 6x
\]
\[
x = 1, \quad y = -2
\]
\[
-2 \cdot \frac{dy}{dx} = 9
\]
\[
\frac{dy}{dx} = -\frac{9}{2}
\]
\[
y + 2 = -\frac{9}{2} (x - 1)
\] or
\[
y = -\frac{9}{2} x + \frac{1}{2}
\]

b) Find the only point on the curve where the tangent line is vertical. (5 pts).

\[
y' = \frac{3x^2 + 6x}{2y}
\]
\[
y' = \frac{0}{2y}
\]
\[
y^2 = 0 = x^3 + 3x^2 = x^2(x + 3)
\]
\[
\text{At } x = 0 : \quad 3x^2 + 6x = 0
\]
\[
\text{At } x = -3 : \quad 3x^2 + 6x = 36 \neq 0
\]

point is: \((-3, 0)\)
PART II: Show the work you did to get the answer in a readable orderly form.

15) In each case below write a formula for $f'(x)$. You don't have to simplify it.

a) $f(x) = \log_2(e^{\cos(x)})$. (3 pts)

$$f'(x) = \frac{1}{\ln(2)} \cdot \frac{1}{e^{\cos(x)}} \cdot e^{\cos(x)} \cdot (-\sin(x))$$

$$= -\frac{1}{\ln(2)} \cdot \sin(x)$$

b) $f(x) = (\sin(x))^{\ln(x)}$. (3 pts)

$$y = \ln(x)$$

$$\ln(y) = \ln(x) \cdot \ln(\sin(x))$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x} \cdot \ln(\sin(x)) + \ln(x) \cdot \frac{\cos(x)}{\sin(x)}$$

$$\frac{dy}{dx} = \sin(x) \ln(x) \left[ \frac{\ln(\sin(x))}{x} + \ln(x) \cdot \cot(x) \right]$$

c) $f(x) = \tan(\sqrt{\cos(x)})$. (4 pts)

$$f'(x) = \sec^2(\sqrt{\cos(x)}) \cdot \frac{1}{2 \sqrt{\cos(x)}} \cdot -\sin(x)$$
PART II : Show the work you did to get the answer in a readable orderly form.

16) A balloon is rising vertically from the ground at a rate of 2 ft/s. A person walking along a straight path at the rate of 3 ft/s passes underneath the balloon at the time that it is 20 ft above the ground. How fast is the distance between the balloon and the walker changing 10 seconds later. (i.e. after passing under the balloon.)

\[ s^2 = x^2 + (y+20)^2 \]

\[ \frac{dx}{dt} = 3 \text{ ft/s} \]

\[ \frac{dy}{dt} = 2 \text{ ft/s} \]

After 10 seconds:

\[ x = 30 \quad y = 20 \]

So

\[ s^2 = 30^2 + (20+20)^2 = 30^2 + 40^2 \]

\[ s = 50 \]

\[ \frac{ds}{dt} = 2x \frac{dx}{dt} + 2(y+20) \frac{dy}{dt} \]

\[ \frac{ds}{dt} = (30)(3) + (40)(2) = 170 \]

\[ \frac{ds}{dt} = \frac{170}{5} \text{ ft/sec} \]