PART I consists of 12 multiple choice questions (worth 5 points each) for a total of 60 points. Mark the correct answer on the answer card. For part I only the answer on the card will be graded.

Part II consists of 4 handgraded problems (worth 10 points each) for a total of 40 points. A correct answer without supporting work may get no credit. Present a readable orderly sequence of steps showing how you got your answer.

Part I (60 points):

1) Find \( \lim_{\theta \to \pi/2} \frac{2\theta - \pi}{\cos(2\pi - \theta)} \).

A) 0
B) 1
C) -2
D) \( \frac{1}{2} \)
E) 2
F) \( \frac{1}{4} \)
G) \( -\frac{3}{4} \)
H) 3
I) \( -\frac{4}{3} \)
J) DNE

2) Find the absolute minimum (y-coordinate) of the curve \( y = \frac{1}{x} + \ln(x) \) on the closed interval \([0.5, 4]\).

A) 1
B) -1.5
C) \( \frac{1}{2} \)
D) -2
E) \( \frac{1}{3} \)
F) -3
G) \( \frac{1}{4} \)
H) -4
I) \( \frac{1}{5} \)
J) 0
3) Find a critical value (x-coordinate) of the function
   \( f(x) = x \sqrt{x^3 + 5} \) (Hint: check domain).

   A) \(-1.7099\)
   B) \(0.6598\)
   C) \(-1.2599\)
   D) \(0\)
   E) \(-0.4768\)
   F) \(0.2342\)
   G) \(-1.8765\)
   H) \(0.2599\)
   I) \(-1.2342\)
   J) No critical values

4) On which of the following intervals is \( f(x) = 6x - x^3 \) increasing?

   A) \((0, 2)\)
   B) \((-2, 0)\)
   C) \((-\sqrt{3}, 0)\)
   D) \((0, \sqrt{3})\)
   E) \((-\sqrt{3}, \sqrt{3})\)
   F) \((-\sqrt{2}, 0)\)
   G) \((0, \sqrt{2})\)
   H) \((-\sqrt{2}, \sqrt{2})\)
   I) \((-2, 2)\)
   J) always decreasing
5) Use the differential at $r = 4$ to approximate the change in the area of a circle when the radius goes from $r = 4$ to $r = 4.3$.

A) $1.2\pi$
B) $1.3\pi$
C) $1.7\pi$
D) $1.8\pi$
E) $2\pi$
F) $2.1\pi$
G) $2.2\pi$
H) $2.3\pi$
I) $2.4\pi$
J) $2.5\pi$

6) Use a linearization at $a = 27$ of $f(x) = \sqrt{x}$ to approximate $\sqrt{25}$.

A) 2.9180
B) 2.9195
C) 2.9205
D) 2.9225
E) 2.9240
F) 2.9259
G) 2.9286
H) 2.9312
I) 2.9365
7) Find the limit, \( \lim_{x \to \infty} \left[ \ln(4x^2 - 2x) - \ln(4 + 3x^2) \right] \).

A) 0.287682  
B) 0.342843  
C) 0.398762  
D) 0.412387  
E) 0.432876  
F) 0.498760  
G) 0.543987  
H) 0.587609  
I) 0.654432  
J) 0.679873

8) Find the coordinates of all the inflection points of \( y = 3x^5 + 5x^4 - 20x^3 \).

A) \( 0, 2, 3 \)  
B) \( -1, 1, 3 \)  
C) \( 1, 2, 4 \)  
D) \( -2, 0, 1 \)  
E) \( -2, 1, 2 \)  
F) \( -1, -\frac{1}{2}, 1 \)  
G) \( -1, 0, 1 \)  
H) \( -\frac{3}{2}, 0, \frac{3}{2} \)  
I) \( -\frac{3}{2}, \frac{3}{2}, \frac{5}{2} \)  
J) no point of inflection
9) Find \( \lim_{x \to \infty} (1 + x)^{\frac{1}{\ln(x)}} \)

A) 0  
B) \(e\)  
C) \(-e\)  
D) \(e^\frac{1}{2}\)  
E) \(e^{-\frac{1}{2}}\)  
F) \(\ln(2)\)  
G) \(\frac{1}{\ln(3)}\)  
H) \(-\infty\)  
I) \(\infty\)  
J) DNE

10) Find where the function \( f(x) = x^4 + 2x^3 \) is concave down.

A) \(x > 1\)  
B) \(0 < x < 1\)  
C) \(x > 3\)  
D) \(1 < x < 3\)  
E) \(x > 2\)  
F) \(0 < x < 2\)  
G) \(x > -1\)  
H) \(-1 < x < 0\)  
I) Everywhere  
J) Nowhere
11) What is the largest possible perimeter for a right triangle with the hypotenuse being \( \sqrt{72} \) in?

A) \( \sqrt{72} + 6 \) in  
B) \( \sqrt{72} + 8 \) in  
C) \( \sqrt{72} + 10 \) in  
D) \( \sqrt{72} + 12 \) in  
E) \( \sqrt{72} + 14 \) in  
F) \( \sqrt{72} + 16 \) in  
G) \( \sqrt{72} + 18 \) in  
H) \( \sqrt{72} + 20 \) in  
I) \( \sqrt{72} + 22 \) in  
J) \( \sqrt{72} + 24 \) in

12) Which of the following is the x-coordinate of a point on the curve \( y = x + \sin(x) \) on the interval \([-2\pi, 2\pi]\) that is a local maximum point.

A) 0  
B) \( \frac{\pi}{2} \)  
C) \( -\frac{\pi}{2} \)  
D) \( \pi \)  
E) \( \frac{3\pi}{2} \)  
F) \( -\frac{3\pi}{2} \)  
G) \( \frac{\pi}{4} \)  
H) \( -\frac{\pi}{4} \)  
I) no local maximum
Name: ____________________________  ID number: ____________________

PART II: Show the work you did to get the answer in a readable orderly form.

13) a) For the function \( f(x) = x^4 - 6x^2 + 4 \), find the all the local max. and local min. points, if any exist. (both x & y coord.) (3 pts)

b) Now find all the inflection points, if any exist. (both x & y coord.) (3 pts)

c) Using the results in parts (a) and (b) sketch a graph of \( f(x) \).
   (Make it clear where the curve is concave up, where concave down.) (4 pts)
PART II: Show the work you did to get the answer in a readable orderly form.

14) The Surface area of a cylinder with an open top is given by the formula
    \[ A = \pi r^2 + 2\pi rh. \]
    Its Volume is \( V = \pi r^2 h \), (\( r \) is radius, \( h \) is height).

   a) If the surface area of this open top cylinder is given as 48 \( \pi \) cm\(^2\),
      find a formula for \( V(r) \), the volume, as a function of \( r \) alone. (3 pts)

   b) Find the largest possible volume for an open top cylinder with a surface area
      of 48 \( \pi \) cm\(^2\). (7 pts)
PART II: Show the work you did to get the answer in a readable orderly form.

15) Consider \( \lim_{x \to \infty} x \cdot \sin \left( \frac{1}{x} \right) \).

a) What type of indeterminate form is it?  (2 pts)

b) Rewrite the limit in a form in which you can use L'Hospital's rule and state what the new form is. (4 pts)

c) Using the form in part (b) solve \( \lim_{x \to \infty} x \cdot \sin \left( \frac{1}{x} \right) \).  (4 pts)
PART II: Show the work you did to get the answer in a readable orderly form.

16) Suppose we start with a cylinder whose height is 10 in. and whose radius is 4 in. We paint the side of the cylinder and with the paint the radius becomes 4.2 in. Since we do not paint the top or bottom the height doesn’t change and we can write a formula for the volume as a function of \( r \), \( V(r) \).

   a) Use a linearization of \( V(r) \) at \( r = 4 \) to approximate the volume of the cylinder after it is painted. (5 pts)

   b) Use the differential \( dV \) at \( r = 4 \) to approximate the volume of paint needed to extend the radius from 4 in to 4.2 in. (5 pts)