1. A spherical balloon is inflated with gas at the rate of $100\pi$ ft$^3$ per minute. In feet per minute, how fast is the radius increasing at the instant the radius is 5 feet. *(The volume of a sphere of radius $r$ is given by $V = \frac{4}{3}\pi r^3$)*

A) 0
B) 1/4
C) 1/3
D) 1/2
E) 3/8
F) 5/8
G) 3/4
H) 1
I) 2
J) None of the above.
2. The length \( L \) of a rectangle is decreasing at \( 2 \) cm/sec while the width \( W \) is increasing at \( 2 \) cm/sec. When \( L = 12 \) cm and \( W = 5 \) cm, how fast (in cm\(^2\)/sec) is the area changing?

A) -16
B) -14
C) -10
D) -6
E) 0
F) 6
G) 10
H) 14
I) 16
J) None of the above.
3. Let \( y = 6 \sinh \frac{x}{3} \). Find \( \frac{dy}{dx} \bigg|_{x=0} \).

A) 0
B) 1/4
C) 1/3
D) 1/2
E) 3/8
F) 5/8
G) 3/4
H) 1
I) 2
J) None of the above.
4. We know that the continuous function \( f(x) = x^2 - 1 \) must take on a minimum value and a maximum value on the closed interval \([-1, 2]\). What are these values? (\( \text{min, max} = \))

A) -1, 0
B) -1, 1
C) -1, 3
D) 0, 1
E) 0, 2
F) 0, 3
G) 1, 3
H) 2, 3
I) 3, 4
J) None of the above.
5. How many roots of the equation \( x^4 + 3x + 1 = 0 \) fall into the closed interval \([-2, -1]\)?

A) 0

B) 1

C) 2

D) 3

E) 4

F) 5

G) 6

H) 7

I) 8

J) None of the above.
6. Find the interval on which \( g(t) = -t^2 - 3t + 3 \) is decreasing

A) \((-\infty, \infty)\)

B) \((-\infty, 0)\)

C) \((-\infty, 3/2)\)

D) \((0, \infty)\)

E) \((-1, \infty)\)

F) \((-3/2, \infty)\)

G) \((3/2, \infty)\)

H) \((3, \infty)\)

I) \((5/2, \infty)\)

J) None of the above.
7. At what $x$ does the absolute minimum of the function $f(x) = x \ln x$ occur? ("$e$" is, as usual, the basis of the natural logarithms.)

A) 0  
B) $1/e$  
C) $2/e$  
D) $e - 1$  
E) 1  
F) $e$  
G) $2e$  
H) $e + 1$  
I) 3e  
J) None of the above.
8. Find all points of inflection of the graph of the function \( f(x) = x^4 - 2x^2 \).

A) 0

B) 1

C) -1

D) -1, 1

E) -1/2, 1/2

F) -1/3, 1/3

G) \(-1/\sqrt{2}, 1/\sqrt{2}\)

H) \(-1/\sqrt{3}, 1/\sqrt{3}\)

I) \(f(x)\) never changes concavity

J) None of the above.
9. What is the smallest perimeter possible for a rectangle whose area is 16 units?

A) 0
B) 6
C) 10
D) 16
E) 25
F) 32
G) 67
H) 71
I) 80
J) None of the above.
10. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by wire fencing. With 800 feet of fence at your disposal, what is the largest area you can enclose? (Give answer in thousands of square feet.)

A) 0

B) 6

C) 10

D) 16

E) 25

F) 32

G) 67

H) 71

I) 80

J) None of the above.
11. Use L'Hospital's rule to find \( \lim_{x \to 0} \frac{1 - \cos x}{x^2} \).

A) 0 
B) 1/4 
C) 1/3 
D) 1/2 
E) 3/8 
F) 5/8 
G) 3/4 
H) 1 
I) 2 
J) None of the above.
12. Use L'Hopital's rule to find $\lim_{x \to 0} (\ln x)^{1/2}$.

A) 0

B) 1/4

C) 1/3

D) 1/2

E) 3/8

F) 5/8

G) 3/4

H) 1

I) 2

J) None of the above.
13. The equation \( x^3 + 3x + 1 = 0 \) has a single real root, which we wish to estimate using Newton’s method. If we start with the initial guess \( x_0 = 0 \), what will be the value of the next approximation \( x_1 \)?

A) 0
B) 1/3
C) -1/3
D) 1/4
E) -1/4
F) 1/5
G) -1/5
H) 1
I) 2
J) None of the above
14. Find \[ \int \left( 1 - \frac{5}{x} \right) dx. \]

A) 0
B) \(-5x + C\)
C) \(x + C\)
D) \(x - 5x\)
E) \(\ln 5x + C\)
F) \(x - 5\ln x + C\)
G) \(x + 5\ln x + C\)
H) \(x - \ln 5x + C\)
I) \(x + \ln 5x + C\)
J) None of the above.
15. Find the general antiderivative of \( f(x) = 7 \sin \frac{x}{3} \).

A) \( C \)

B) \( 7 \cos \frac{x}{3} + C \)

C) \( -7 \cos \frac{x}{3} + C \)

D) \( \frac{7}{3} \cos \frac{x}{3} + C \)

E) \( -\frac{7}{3} \cos \frac{x}{3} + C \)

F) \( 21 \cos \frac{x}{3} + C \)

G) \( -21 \cos \frac{x}{3} + C \)

H) \( x - 21 \cos \frac{x}{3} + C \)

I) \( x + \cos \frac{7x}{3} + C \)

J) None of the above.
Hand graded section. Six points/problem + 1 point for writing your name *LEGIBLY*. Show your work for partial credit. Use back of this sheet if necessary.

A rectangle with sides $a$ and $b$ has perimeter 4. What values should $a$ and $b$ have to make the area of the rectangle a maximum?
1) Express the area as $A(a)$ for $a$ in some closed interval.
2) Use this information to find the maximum value of $A$.
3) Use the first derivative test to verify that this is a maximum.
4) Now do the same using the second derivative test.