Calculus I

Math 131 — Spring 2008

Exam 3, April 8

Name: 

Student-ID: 

This exam contains sixteen problems. Problems 1 - 14 are multiple choice problems, which each count 5% towards your total score. Problem 15 and 16 will be hand-graded (with a possibility of partial credit) and count 15% each towards your total score.

Problem 1

On which intervals is \( f(x) = 4x^3 - x^4 \) increasing?

A) Only on \((-3, 0)\)
B) On \((-3, 0)\) and \((0, \infty)\)
C) On \((-\infty, 0)\) and \((0, 3)\)
D) Only on \((0, 3)\)
E) On \((-\infty, -\sqrt{3})\) and \((-\sqrt{3}, \sqrt{3})\)
F) Only on \((-\sqrt{3}, \sqrt{3})\)
G) Only on \((3, \infty)\)
H) This function is nowhere increasing.
Problem 2

List the x-coordinates of all the inflection points of

\[ y = \frac{1}{3}x^6 + \frac{1}{2}x^5 - 5x^4 + 4x + 8 \]

A) -6, 0, 1
B) -6, 0
C) -3, 0, 2
D) 0, 2
E) -3, 2
F) -2, 0, 3
G) -2, 0
H) -2, 3
Problem 3

Compute

\[ \lim_{x \to 0} \frac{x - \sin x}{x^3} \]

A) $-1$
B) $-\frac{1}{3}$
C) $-\frac{1}{6}$
D) 0
E) $\frac{1}{6}$
F) $\frac{1}{3}$
G) 1
H) 6
Problem 4

Find the absolute minimum value and absolute maximum value of the function \( f(x) = x - 2\sin x \) on the interval \([0, \pi]\).

\[(\text{min}, \text{max}) = (\cdots, \cdots)\]

A) \(\left(\frac{\pi}{3}, 0\right)\)
B) \(\left(\pi, \sqrt{3}\right)\)
C) \((0, \frac{\pi}{3} - \sqrt{3})\)
D) \((\pi - \sqrt{3}, \frac{1}{3})\)
E) \((\frac{1}{3}, 0)\)
F) \((0, \pi)\)
G) \(\left(\frac{\pi}{3} - \sqrt{3}, \pi\right)\)
H) \((0, \frac{\pi}{3})\)
Problem 5

If $f$ is a differentiable function on the interval $[3, 7]$ and if $f(3) = 1$ and $f(7) = 4$, then the Mean Value Theorem says that there is some number $c$ in between 3 and 7 such that $f'(c) = \ldots$

Calculate $f'(c)$ in this situation.

A) $\frac{3}{4}$
B) $\frac{4}{3}$
C) $\frac{1}{4}$
D) $\frac{3}{2}$
E) $\frac{7}{4}$
F) 4
G) 0
H) 1
Problem 6

The product of two positive numbers $x$ and $y$ equals 24. Find the smallest number that their sum can be.

A) $\sqrt{24}$
B) 6
C) $2\sqrt{12}$
D) 8
E) $2\sqrt{24}$
F) 11
G) 12
H) 24
Problem 7

Find all vertical asymptotes of

\[ f(x) = \frac{x^2 - 4x + 3}{x^2 - 5x + 6} \]

A) \( x = 1 \)
B) \( x = 2 \)
C) \( x = 2, x = 3 \)
D) \( x = 1, x = 3 \)
E) \( x = 1, x = 2 \)
F) \( x = 3 \)
G) \( x = 1, x = 2, x = 3 \)
H) There are no vertical asymptotes.
Problem 8

If a cylinder has height $h$, and the radius of the base circle is $r$, then the volume equals $\pi r^2 h$, and the total surface area equals $2\pi rh + 2\pi r^2$.

For a cylinder with a volume of 16 cm$^3$, determine the radius $r$ (in cm) which minimizes the total surface area.

A) $\frac{\sqrt{\pi}}{2}$  
B) $\frac{\sqrt{\pi}}{3}$  
C) $\frac{2\pi}{3}$  
D) $\frac{3}{2\pi}$  
E) $2\sqrt{\pi}$  
F) $\frac{5}{\sqrt{\pi}}$  
G) $3\sqrt{\pi}$  
H) $\frac{5}{\sqrt{3}}$
Problem 9

\[ \int \left( 5x^2 - 17x^{-\frac{1}{2}} + e^x \right) \, dx = \cdots \]

A) \( \frac{5}{3}x^3 - 34x^{\frac{1}{2}} + e^x + C \)
B) \( \frac{5}{3}x^3 - 34x^{\frac{1}{2}} - e^x + C \)
C) \( \frac{5}{3}x^3 - \frac{17}{2}x^{\frac{1}{2}} + e^x + C \)
D) \( \frac{5}{3}x^3 - \frac{17}{2}x^{\frac{1}{2}} - e^x + C \)
E) \( 5x^3 - 34x^{\frac{1}{2}} + e^x + C \)
F) \( 5x^3 - 34x^{\frac{1}{2}} - e^x + C \)
G) \( 5x^3 - \frac{17}{2}x^{\frac{1}{2}} + e^x + C \)
H) \( 5x^3 - \frac{17}{2}x^{\frac{1}{2}} - e^x + C \)
Problem 10

Calculate the limit:

\[
\lim_{x \to -\infty} \frac{x^2 \sqrt{4 - e^{-x}}}{-2x^2 + 5x - 3}
\]

A) \(\infty\)
B) \(-\infty\)
C) 0
D) -1
E) 1
F) \(-\frac{1}{2}\)
G) -2
H) \(-\frac{\sqrt{3}}{2}\)
Problem 11

On which intervals is \( f(x) = (x^2 - 4x + 2)e^x \) concave down?

A) \((-\infty, 1 - \sqrt{3})\) and \((1 + \sqrt{3}, \infty)\)
B) \((1 - \sqrt{3}, 1 + \sqrt{3})\)
C) \((-1, 1)\)
D) \((-2, 2)\)
E) \((-\infty, -2)\) and \((2, \infty)\)
F) \((-\infty, 2 - \sqrt{2})\) and \((2 + \sqrt{2}, \infty)\)
G) \((2 - \sqrt{2}, 2 + \sqrt{2})\)
H) \((-\sqrt{2}, \sqrt{2})\)
Problem 12

\[
\lim_{{t \to 1}} \frac{e^{2t-2} - \cos(1-t) - 2t + 2}{t^2 - 2t + 1} = \ldots
\]

A) $-1$
B) $-\frac{1}{2}$
C) 0
D) $\frac{1}{2}$
E) 1
F) $\frac{3}{2}$
G) 2
H) $\frac{5}{2}$
Problem 13

What are the $x$-values of the critical points of $f(x) = \sqrt{4 - (x - 1)^2}$?

A) $x = -2, x = 1, x = 2$
B) $x = -2, x = 2$
C) $x = 1$
D) $x = 1, x = 2, x = 3$
E) $x = -1, x = 3$
F) $x = 1$
G) $x = -1, x = 1, x = 3$
H) There are no critical points.
Problem 14

Consider the function $f(x) = x + \frac{4}{(x-3)^2}$ with domain $(2, 5]$.
Does $f(x)$ have an absolute minimum value? If so, where does it occur?
What about an absolute maximum value?

A) min at $x = 5$, no max
B) min at $x = 5$, max at $x = 3$
C) min at $x = 4$, no max
D) min at $x = 4$, max at $x = 3$
E) max at $x = 2$, no min
F) max at $x = 2$, min at $x = 4$
G) max at $x = 3$, no min
H) max at $x = 2$, min at $x = 5$
Written problem - Show your work

You can get partial credit for this one, up to a maximum of 14 points + 1 point for writing down your name and student ID legibly!

Problem 15

Sketch the graph of \( f(x) = x^4 - x^3 \).
(Point out at least the following by calculation: critical points, where is \( f \) increasing/decreasing, inflection points, where is \( f \) concave up/down. Then make a sketch of the graph: the parts where \( f \) is increasing/decreasing/concave up/concave down should be clearly visible. Point out where the critical points and/or inflection points are located.)
Written problem - Show your work

You can get partial credit for this one, up to a maximum of 14 points + 1 point for writing down your name and student ID legibly!

Problem 16

Approximate a root of \( f(x) = 2x + \ln x \) using Newton’s Method, with initial value \( x_0 = 1 \). Do this by computing the first three approximations \( x_1, x_2 \) and \( x_3 \), using at least 3 decimals when rounding off numbers in your calculations.