1. Which of the following are critical points of the function $f(x) = 1 - (x-1)^{2/3}$?

A) 0

B) 1

C) 5/4

D) 7/3

E) $\sqrt{2}$

F) $\sqrt{3}$

G) π

H) 2π

I) 3π

J) None of the above

$\cancel{f''(x) = 2(3x^2+1)}$

$$f'(x) = \frac{-2}{3(x-1)^{1/3}}$$

f' IS NEVER = 0 WHERE DEFINED,
BUT f'(1) IS NOT DEFINED.
SO x = 1 IS C.P.

2. The continuous function $f(x) = x^3 - 12x^2 + 21x$ attains a maximum value on the closed interval $[0, 2]$. At which x does this occur?

A) 0

B) 1

C) 5/4

D) 7/3

E) $\sqrt{2}$

F) $\sqrt{3}$

G) π

H) 2π

I) 3π

J) None of the above

$f'(x) = 3x^2 - 24x + 21$

SO $0 = f'(x) \Leftrightarrow$

$0 = x^2 - 8x + 7 = (x-1)(x-7)$

C.P. ~~ARE~~ IN $[0, 2]$ ARE: 1

	x	$f(x)$
E.P	0	0
C.P.	1	10 ← MAX AT $x = 1$
E.P.	2	2

3. Find the maximum value taken on by $f(x) = \sin x \cos x$ on the interval $[0, \pi/2]$.

A) 0

B) 1/3

C) 1/2

D) $1/\sqrt{2}$

E) $1/\sqrt{3}$

F) $1/\pi$

G) $1/2\pi$

H) $\pi/3\sqrt{2}$

I) $\pi/2\sqrt{3}$

J) None of the above

$f'(x) = 2\cos^2 x - 1$

$0 = f'(x) \Leftrightarrow \cos^2 x = \frac{1}{2}$

$\Leftrightarrow \cos x = \frac{1}{\sqrt{2}} \Leftrightarrow x = \pi/4$

	x	$f(x)$
E.P.	0	0
C.P.	$\pi/4$	$1/2$ $\leftarrow$ MAX $= f(\pi/4) = 1/2$
E.P.	$\pi/2$	0

4. Consider the function $f(x) = e^x - x$ on the interval [-1, 1]. Find a point c in the interval which satisfies the conclusion of the mean value theorem for this f(x) and [a, b]. c =

A) 0

B) .0082

C) .0132

D) .1126

E) .1375

F) .1614

G) .1853

H) .2054

I) .2709

J) None of the above

MVT: EXISTS c SO THAT

$$(*) \quad f'(c) = \frac{f(b) - f(a)}{b - a}.$$

$f(x) = e^x - x$, $a = -1$, $b = 1$.

$f'(x) = e^x - 1$ so

$$(*) \quad e^c - 1 = \frac{(e-1) - (e^{-1}+1)}{1-(-1)}$$

$$= \frac{e - e^{-1} - 2}{2} = \frac{e - e^{-1}}{2} - 1$$

$$\text{so } e^c = \frac{e - e^{-1}}{2}$$

$$\Rightarrow c = \ln\left(\frac{e - e^{-1}}{2}\right) \approx 0.1614$$

5. The finction $f(x) = 3x^5 - 5x^3$ is decreasing on an interval [-1, a] and then is increasing on [a, ∞]. Find a.

A) 0

B) 1

C) 5/4

D) 7/3

E) $\sqrt{2}$

F) $\sqrt{3}$

G) π

H) 2π

I) 3π

J) None of the above

$f'(x) = 15x^4 - 15x^2$ so

C.P. where $0 = x^4 - x^2 = x^2(x+1)(x-1)$

so C.P. at. $-1, 0, 1$

NOTE $f' < 0$ on $(-1, 0)$ AND $(0, 1)$

AND $f' > 0$ on $(1, \infty)$ $\Rightarrow a = 1$

6. $f(x) = x^{5/3}$ changes concavity at only one point a. Find a.

A) 0

B) 1/3

C) 1/2

D) $1/\sqrt{2}$

E) $1/\sqrt{3}$

F) $1/\pi$

G) $1/2\pi$

H) $\pi/3\sqrt{2}$

I) $\pi/2\sqrt{3}$

J) None of the above

$f''(x) = \frac{10}{9} x^{-1/3}$

$f'' \neq 0$ WHERE DEFINED.

f'' NOT DEFINED AT $x = 0$

BUT : $x < 0 \Rightarrow f''(x) < 0 \Rightarrow$ c. d.

$x > 0 \Rightarrow f''(x) > 0 \Rightarrow$ c. u.

$a = 0$.

7. the function $f(x) = \frac{1}{x^2+1}$ has a single point of inflection in the interval $(0, \infty)$. Find it.

$$f''(x) = \frac{2(3x^2-1)}{(x^2+1)^3}$$

CLEARLY, $f''(x) = 0$

$\Longleftrightarrow 3x^2 - 1 = 0 \Longleftrightarrow$

$x = \pm \frac{1}{\sqrt{3}}$, so $x = \frac{1}{\sqrt{3}}$

A) 0

B) 1/3

C) 1/2

D) $1/\sqrt{2}$

E) $1/\sqrt{3}$

F) $1/\pi$

G) $1/2\pi$

H) $\pi/3\sqrt{2}$

I) $\pi/2\sqrt{3}$

J) None of the above

8. the function $f(x) = e^{-x^2}$ has a single point of inflection in the interval $(0, \infty)$. Find it.

A) 0

B) 1/3

C) 1/2

D) $\frac{1}{\sqrt{2}}$

E) $\frac{1}{\sqrt{3}}$

F) $\frac{1}{\pi}$

G) $\frac{1}{2\pi}$

H) $\frac{\pi}{3\sqrt{2}}$

I) $\frac{\pi}{2\sqrt{3}}$

J) None of the above

$f''(x) = (4x^2 - 2) e^{-x^2}$

so $f''(x) = 0 \Leftrightarrow$

$4x^2 - 2 = 0$

$\Leftrightarrow x = \pm \frac{1}{\sqrt{2}}$, so $x = \frac{1}{\sqrt{2}}$

9. Find any points x at which the function $f(x) = \frac{x^2}{x^2 - 5x + 6}$ has a vertical asymptote.

A) 1

B) 2

C) 3

D) -2

E) -3

F) 2 and 3

G) 1 and 2

H) -1 and 3

I) -2 and -3

J) None of the above

NOTE $f(x) = \frac{x^2}{(x-2)(x-3)}$

$\lim_{x \to 2^+} f(x) = -\infty$, $\lim_{x \to 2^-} f(x) = +\infty$.

V.A. AT $x = 2$

$\lim_{x \to 3^-} f(x) = -\infty$, $\lim_{x \to 3^+} f(x) = +\infty$

V.A. AT $x = 3$

10. Find any lines y = b which are horizontal asymptotes of $f(x) = \frac{x^2}{x^2 - 5x + 6}$. b =

A) 0

B) 1

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{1}{1 - \frac{5}{x} + \frac{6}{x^2}} = 1$$

C) 5/4

D) 7/3

E) $\sqrt{2}$

F) $\sqrt{3}$

G) π

H) 2π

I) 3π

J) None of the above

11. Find the x – coordinate of the point on the curve $y = \sqrt{x}$ which is closest to the point (1, 0).

A) 0

B) 1/3

C) 1/2

D) $1/\sqrt{2}$

E) $1/\sqrt{3}$

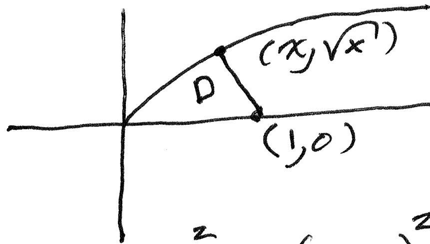
F) $1/\pi$

G) $1/2\pi$

H) $\pi/3\sqrt{2}$

I) $\pi/2\sqrt{3}$

J) None of the above



$$D^2 = (x-1)^2 + (\sqrt{x} - 0)^2$$

$$D^2 = x^2 - x + 1$$

ENUF TO MIN D^2:

$$\frac{dD^2}{dx} = 2x - 1 \Rightarrow \underline{x = \tfrac{1}{2}}$$

$$\frac{d^2D^2}{dx^2} = 2 > 0 \Rightarrow \text{MIN}$$

12. A box will be constructed from two kinds of sheet metal. The metal for the top and bottom, which are to be square, costs \$1 per m^2. The metal for the sides costs \$2 per m^2. *To the nearest dollar,* what is the material cost for the cheapest such box whose volume is 20 m^3 ?

A) 16

B) 43

C) 70

D) 124

E) 467

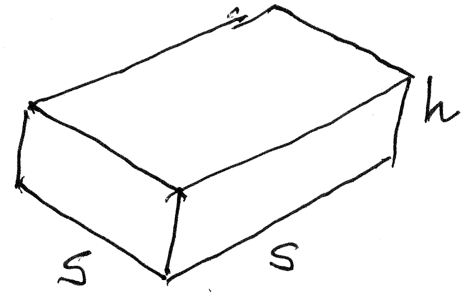
F) 1085

G) 5238

H) 11664

I) 15893

J) None of the above



$20 = V = s^2 h$ SO $h = \frac{20}{s^2}$

TOTAL AREA OF SIDES

$= 4sh = 4s\left(\frac{20}{s^2}\right) = \frac{80}{s}$

TOTAL AREA OF TOP, BOTTOM

$= 2s^2$

$C(s) = 1(2s^2) + 2\left(\frac{80}{s}\right)$

$C(s) = 2s^2 + \frac{160}{s}$

$C'(s) = 4s - \frac{160}{s^2}$

$0 = C'(s) \iff 0 = s - \frac{40}{s^2} \iff s^3 = 40$

$s = 2\sqrt[3]{5}$

$C''(2\sqrt[3]{5}) > 0 \Rightarrow$ MIN

$C(2\sqrt[3]{5}) \cong \$70.18$

13. What is the maximum possible area of a right triangle the length of whose legs add up to 2?

A) 0

B) 1/3

C) 1/2

D) $1/\sqrt{2}$

E) $1/\sqrt{3}$

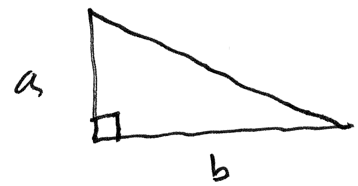
F) $1/\pi$

G) $1/2\pi$

H) $\pi/3\sqrt{2}$

I) $\pi/2\sqrt{3}$

J) None of the above



$A = \frac{1}{2}ab$, $a+b=2 \Rightarrow b=2-a$

$A(a) = \frac{1}{2}a(2-a)$

$A(a) = a - \frac{1}{2}a^2$

$\cancel{A'(a) = a - \frac{1}{2}a^2}$

~~For MAX A' = 0~~ $\cancel{0 = 1 - \frac{1}{2}a}$

$0 = A'(a) = 1-a \Rightarrow a=1 \Rightarrow b=1$

$A''(a) = -1 < 0 \therefore$ MAX

$A = \frac{1}{2}\cdot 1 \cdot 1 = \underline{\frac{1}{2}}$

14. According to U. S. Postal regulations, a rectangular box is "oversized" if the sum of its height and its girth (perimeter of the base) exceeds 108 inches. Find the maximum volume (in cubic inches) that a box can have if 1) it has a square base, and 2) it is *not* oversized.

A) 16

B) 43

C) 70

D) 124

E) 467

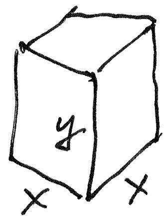
F) 1085

G) 5238

H) 11664

I) 15893

J) None of the above



$V = x^2 y$

$108 = 4x + y$

$\Rightarrow y = 108 - 4x$

$V(x) = x^2(108-4x) = 108x^2 - 4x^3$

$0 = V'(x) = 216x - 12x^2$

$\Leftrightarrow (x = 0)$ NOT MAX

OR $216 = 12x$

$x = 18$

SO $\cancel{y = 108 - 4(18)}$.

$V(18 = 108(18)^2 - 4(18)^3$

$= 11\,664$

15. Evaluate $\lim_{x\to 0^+}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$. $= L$

A) 0

B) 1/3

C) 1/2

D) $1/\sqrt{2}$

E) $1/\sqrt{3}$

F) $1/\pi$

G) $1/2\pi$

H) $\pi/3\sqrt{2}$

I) $\pi/2\sqrt{3}$

J) None of the above

$$L = \lim_{x\to 0^+} \frac{x-\sin x}{x\sin x} \quad \left(\frac{0}{0}\right)$$

$$\text{so} \stackrel{L.H.}{=} \lim_{x\to 0^+} \frac{1-\cos x}{\sin x + x\cos x} \quad \left(\frac{0}{0}\right)$$

$$\stackrel{L.H.}{=} \lim_{x\to 0^+} \frac{\sin x}{2\cos x - x\sin x} = \frac{0}{2} = 0$$

16. Find $\lim\limits_{x \to 1} \dfrac{\tan^{-1} x - \pi/4}{\tan\left(\frac{\pi}{4}x\right) \cancel{+} 1}$. $= L$

A) 0

B) 1/3

C) 1/2

D) $1/\sqrt{2}$

E) $1/\sqrt{3}$

F) $1/\pi$

G) $1/2\pi$

H) $\pi/3\sqrt{2}$

I) $\pi/2\sqrt{3}$

J) None of the above

L " $= \frac{0}{0}$ " SO BY L'H. RULE:

$$L = \lim_{x \to 1} \frac{\frac{1}{x^2+1}}{\frac{\pi}{4}\sec^2\left(\frac{\pi}{4}x\right)}$$

$$= \frac{1/2}{\pi/4 \, (\sqrt{2})^2} = \frac{1}{\pi}$$

17. Suppose Newton's method is used to approximate a root of $x^4 + x^2 - 2x - 1 = 0$. The initial guess is $x_o = 1$. Find the next approximation x_1.

A) 0

B) 1

C) 5/4

D) 7/3

E) $\sqrt{2}$

F) $\sqrt{3}$

G) π

H) 2π

I) 3π

J) None of the above

$f(x) = x^4 + x^2 - 2x - 1$

$f'(x) = 4x^3 + 2x - 2$

SO ITER REL'N IS:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \frac{f(1)}{f'(1)} = 1 - \frac{-1}{4} = \frac{5}{4}$$

18. Evaluate $\lim_{x\to\infty}\frac{3x^6+4x^2-7}{9x^6+12x^4+2x}$. $= L$

DIVIDE NUM. & DEN. BY x^6:

$$L = \lim_{x\to\infty} \frac{3+\frac{4}{x^4}-\frac{7}{x^6}}{9+\frac{12}{x^2}+\frac{2}{x^5}} = \frac{3}{9} = \frac{1}{3}$$

A) 0

B) 1/3

C) 1/2

D) $1/\sqrt{2}$

E) $1/\sqrt{3}$

F) $1/\pi$

G) $1/2\pi$

H) $\pi/3\sqrt{2}$

I) $\pi/2\sqrt{3}$

J) None of the above

19. Let y(t) be the solution to the initial value problem $\frac{dy}{dt} = 5 - 2t^2$, $y(1) = \frac{20}{3}$.

Find y(0).

A) 0

B) 1

C) 5/4

D) 7/3

E) $\sqrt{2}$

F) $\sqrt{3}$

G) π

H) 2π

I) 3π

J) None of the above

$$\frac{dy}{dt} = 5 - 2t^2$$

so $y(t) = 5t - \frac{2}{3}t^3 + C$

USE IC :

$$\frac{20}{3} = y(1) = 5 \cdot 1 - \frac{2}{3} 1^3 + C$$

$$\Rightarrow C = \frac{7}{3}$$

so $y(t) = 5t - \frac{2}{3}t^3 + \frac{7}{3}$

$$y(0) = \frac{7}{3}$$

20. Let y(x) be the solution of $\frac{dy}{dx} = \sec^2 3x$, $y\left(\frac{\pi}{4}\right) = 2$. Find y(0).

A) 0

B) 1

C) 5/4

D) 7/3

E) $\sqrt{2}$

F) $\sqrt{3}$

G) π

H) 2π

I) 3π

J) None of the above

$$\frac{dy}{dx} = \sec^2 3x$$

So $y(x) = \frac{1}{3}\tan 3x + C$

I.C.

$2 = y\left(\frac{\pi}{4}\right) = \frac{1}{3}\tan\frac{3\pi}{4} \cdot C$

$2 = \frac{1}{3}(-1) + C$

$\Rightarrow C = \frac{7}{3}$

$\therefore y(x) = \frac{1}{3}\tan 3x + \frac{7}{3}$

$y(0) = 0 + \frac{7}{3}$

$y(0) = \frac{7}{3}$