This exam should have 18 questions. Part I will have 16 multiple choice questions, 5 points each. Part II will have 2 hand graded questions, 10 points each. Please check to see that your exam is complete. If you do not have a PENCIL to mark your card, please ask to borrow one from your proctor.

Write your ID number (not your SS number) on the six blank lines at the top of your answer card, using one blank for each digit. **Shade in the corresponding boxes below.** Also print your name at the top of your card. If your card becomes damaged please ask your proctor for a new one.

You may **only** use a scientific calculator (**not** one with graphing). You may bring with you a 3x5 card with any data on it, together with the trigonometry info. found on the syllabus.

**PART I** : (80 points)

1. Given that \( \frac{dy}{dx} = \frac{1}{2} e^{2x} + \frac{1}{2} \) and that \( y(0) = 1 \), find \( y(1) \).

   A) \( \frac{7}{3} e^{\frac{3}{2}} + \frac{1}{3} \)  B) \( \frac{2e^{2}+3}{4} \)  C) \( \frac{1}{2} e^{2} + \frac{2}{3} \)  D) \( \frac{e^{2}-8}{3} \)  E) \( \frac{2}{5} e^{3} + \frac{1}{3} \)

   F) \( \frac{1}{2} e^{3} + 2 \)  G) \( \frac{e^{2}+5}{4} \)  H) \( \frac{1}{5} e^{2} + \frac{3}{2} \)  I) \( \frac{e^{2}+6}{4} \)  J) \( \frac{1}{2} e^{3} + \frac{1}{2} \)
2. Approximate the area under the curve $y = \frac{1}{x}$, $1 \leq x \leq 3$, using the left point Riemann sum with $n = 4$.

A) 1.0986  
B) 1.1136  
C) 1.1478  
D) 1.1665  
E) 1.1879  
F) 1.2135  
G) 1.2472  
H) 1.2651  
I) 1.2833  
J) 1.3091

3) For $y = \int_{1}^{x^3} \sqrt{t} \, dt$, find the value of $\frac{dy}{dx}$ at $x = 2$.

A) 12  
B) 24  
C) 32  
D) 44  
E) 15  
F) 36  
G) 47  
H) 28  
I) 39  
J) 33
4. Evaluate \( \int_{0}^{\ln(2)} e^{3x} \, dx \).

A) \( \frac{2}{3} \)

B) \( \frac{3}{4} \)

C) \( \frac{4}{3} \)

D) \( \frac{3}{4} \)

E) \( \frac{7}{3} \)

F) \( \frac{4}{5} \)

G) \( \frac{9}{5} \)

H) \( \frac{7}{4} \)

I) \( \frac{9}{2} \)

J) \( \frac{9}{4} \)

5) Evaluate \( \int_{0}^{a} x \sqrt{a^2 - x^2} \, dx \) using the substitution \( u = a^2 - x^2 \) (a is some constant).

A) 0

B) a

C) \( a^2 \)

D) \( a^3 \)

E) \( \frac{a}{2} \)

F) \( \frac{a^2}{3} \)

G) \( \frac{a^3}{3} \)

H) \( \frac{a^4}{5} \)

I) \( a^2 - a \)
6. Evaluate $\int_{0}^{3} \tan^2(x) \sec^4(x) \, dx$ using the substitution $u = \tan(x)$. (Hint: Also needs a trig. identity)

A) $\frac{1}{15}$
B) $\frac{2}{15}$
C) $\frac{1}{5}$
D) $\frac{4}{15}$
E) $\frac{1}{3}$
F) $\frac{5}{6}$
G) $\frac{7}{15}$
H) $\frac{8}{15}$
I) $\frac{3}{5}$
J) $\frac{2}{3}$

7) Evaluate the indefinite integral $\int \frac{1}{x^2} \sin\left(\frac{1}{x}\right) \cos\left(\frac{1}{x}\right) \, dx$.

A) $\frac{1}{2} \cos\left(\frac{1}{x}\right) + C$
B) $\frac{1}{2} \sin\left(\frac{1}{x}\right) + C$
C) $\frac{1}{2} \cos^2\left(\frac{1}{x}\right) + C$
D) $\sin\left(\frac{1}{x}\right) \cos\left(\frac{1}{x}\right) + C$
E) $2 \cos^2\left(\frac{1}{x}\right) + C$
F) $\sin^2\left(\frac{1}{x}\right) \cos^2\left(\frac{1}{x}\right) + C$
G) $2 \sin\left(\frac{1}{x}\right) + C$
H) $4 \cos^2\left(\frac{1}{x}\right) + C$
I) $2 \sin\left(\frac{1}{x}\right) \cos\left(\frac{1}{x}\right) + C$
8. Evaluate \( \int_4^8 \frac{\log_2(x)}{x} \, dx \).
A) 1.7329
B) 1.8112
C) 2.1357
D) 2.5481
E) 2.9765
F) 3.3652
G) 3.5672
H) 3.7563
I) 3.9856

9. Find the area in the first quadrant that is bounded by the curve \( y = \frac{1}{4}x^2 \), the line \( y = x \) and the line \( y = 1 \).
A) \( \frac{1}{15} \)
B) \( \frac{2}{15} \)
C) \( \frac{1}{5} \)
D) \( \frac{4}{15} \)
E) \( \frac{1}{3} \)
F) \( \frac{5}{6} \)
G) \( \frac{7}{15} \)
H) \( \frac{3}{15} \)
I) \( \frac{3}{5} \)
J) \( \frac{2}{3} \)
10. Find the area of the region enclosed by $y = x^3$ and $y = x$, $0 \leq x \leq 2$ (Hint: you'll need 2 integrals).

A) $\frac{1}{2}$

B) $\frac{3}{2}$

C) $\frac{5}{2}$

D) $\frac{7}{2}$

E) $\frac{9}{2}$

F) $\frac{11}{2}$

G) $\frac{13}{2}$

H) $\frac{9}{4}$

I) $\frac{17}{4}$

J) $\frac{19}{4}$
11. Find the volume of the solid obtained by rotating the region bounded by the lines, $y = 3x$, $y = 6$ and $x = 0$ about the $y$-axis.

A) $2\pi$

B) $\frac{8}{3}\pi$

C) $4\pi$

D) $\frac{13}{2}\pi$

E) $5\pi$

F) $\frac{17}{3}\pi$

G) $6\pi$

H) $\frac{20}{3}\pi$

I) $8\pi$

J) $\frac{27}{2}\pi$
12. Find the volume of the solid whose base is the region in the x-y plane bounded by the circle $x^2 + y^2 = 9$ and whose cross-sections perpendicular to the x-axis are squares.

A) 120
B) 144
C) 162
D) 184
E) 206
F) 220
G) 246
H) 260
I) 284
J) 300
13) Consider the solid obtained by rotating about the y-axis the region in the first quadrant bounded by the curve \( y = x^3 \) and the lines \( x = 0 \) and \( y = 8 \). Write out the integral you would get, if you want to calculate the volume of the solid using the shell method.

A) \( \pi \int_0^8 y^{2/3} \, dy \)

B) \( \pi \int_0^2 (4-y^{2/3}) \, dy \)

C) \( \pi \int_0^8 x^6 \, dx \)

D) \( \pi \int_0^2 (8-x^6) \, dx \)

E) \( 2\pi \int_0^8 (8-x^3) \, dx \)

F) \( 2\pi \int_0^2 (8-x^3) \, dx \)

G) \( 2\pi \int_0^8 (8x-x^4) \, dx \)

H) \( 2\pi \int_0^2 (8x-x^4) \, dx \)

I) \( 2\pi \int_2^8 y^{2/3} \, dy \)

J) \( 2\pi \int_2^8 (4-y^{2/3}) \, dy \)
14) Find the length of the curve \( y = \frac{1}{13} x^3 + \frac{1}{x} \), for \( 1 \leq x \leq 3 \).

(Hint: \( 1 + (y')^2 \) is a perfect square.)

A) \( \frac{1}{2} \)
B) \( \frac{3}{4} \)
C) \( \frac{5}{2} \)
D) \( \frac{12}{5} \)
E) \( \frac{17}{6} \)
F) \( \frac{21}{8} \)
G) \( \frac{25}{3} \)
H) \( \frac{31}{6} \)
I) \( \frac{37}{5} \)
J) \( \frac{38}{3} \)
15) Find the **area of the surface** generated by revolving the curve \( x = t, \ y = \frac{1}{2}t^2, \ 0 \leq t \leq \sqrt{3} \), about the **y-axis**.

A) \( \frac{\pi}{3} \)

B) \( \frac{4\pi}{3} \)

C) \( \frac{5\pi}{3} \)

D) \( \frac{7\pi}{3} \)

E) \( \frac{10\pi}{3} \)

F) \( \frac{10\pi}{5} \)

G) \( \frac{12\pi}{7} \)

H) \( \frac{12\pi}{5} \)

I) \( \frac{14\pi}{5} \)

J) \( \frac{14\pi}{3} \)
16) The integral you would get if you wanted to compute the area of the surface you would get by rotating the curve \( y = \sqrt[3]{x} \) for \( 1 \leq x \leq 8 \) about the y-axis is:

A) \( 2\pi \int_{1}^{2} x^3 \sqrt{1 + 9x^4} \, dx \)

B) \( 2\pi \int_{1}^{8} x^2 \sqrt{1 + 9x^2} \, dx \)

C) \( 2\pi \int_{1}^{2} x \sqrt{1 + 4x^2} \, dx \)

D) \( 2\pi \int_{1}^{8} x^3 \sqrt{1 + 9x^4} \, dx \)

E) \( 2\pi \int_{1}^{2} \sqrt{1 + 9x^4} \, dx \)

F) \( 2\pi \int_{1}^{8} y^2 \sqrt{1 + 9y^3} \, dy \)

G) \( 2\pi \int_{1}^{2} y \sqrt{1 + 4y^2} \, dy \)

H) \( 2\pi \int_{1}^{8} y^3 \sqrt{1 + 9y^4} \, dy \)

I) \( 2\pi \int_{1}^{2} \sqrt{1 + 9y^4} \, dy \)

J) \( 2\pi \int_{1}^{2} y^3 \sqrt{1 + 9y^4} \, dy \)
Part II: (20% of test points) In each problem, clearly show your solution in the space provided. Present a sequence of steps showing how you got the answer.

**PLEASE PUT YOUR NAME AND ID NUMBR ON EACH OF THE SHEETS OF PART II** (IN CASE THE SHEETS GET SEPERATED).

17) Solve the following integrals (5 points each):

a) \( \int \frac{x^2}{x^4 + x} \, dx \quad (x > 0) \)

b) \( \int \frac{1}{x^2} \sqrt{1 + \frac{1}{x}} \, dx \)
Name ________________________________
ID number __________________________

Part II: (40% of test points) In each problem, clearly show your solution in the space provided. Present a sequence of steps showing how you got the answer.

Please put your name and ID numbr on each of the sheets of Part II (in case the sheets get separated).

18) Consider the solid generated by revolving the region in the first quadrant, enclosed by the curve \( y = x^2 \) and the line \( y = 2x \), around the y-axis. (5 points each)

a) Find the volume of this solid using the washer method.

c) Find the volume of this solid using the shell method.