This exam contains 16 multiple choice questions and 2 hand graded questions. The multiple choice questions are worth 5 points each and the hand graded questions are worth a total of 20 points. The latter questions will be evaluated not only for having the correct solutions but also for clarity. Points may be taken off for confusing and disorganized writing, even when the answer is correct.

1) Find the volume of the solid obtained by rotating the region in the first quadrant enclosed by the graphs $y = x^3$, $y = x^{\frac{1}{3}}$, about the $y$-axis.

A) $\frac{3\pi}{4}$
B) $\frac{3\pi}{2}$
C) $\frac{32\pi}{3}$
D) $\frac{9\pi}{8}$
E) $\frac{7\pi}{3}$
F) $\frac{16\pi}{15}$
G) $\frac{16\pi}{7}$
H) $\frac{3\pi}{24}$
I) $\frac{3\pi}{32}$
J) $\frac{20\pi}{33}$

2) Find the volume of the solid obtained by rotating the region enclosed by the parabola $y = x - x^2$ and the x-axis about the vertical line $x = -1$.

A) $\frac{3\pi}{4}$
B) $\frac{1\pi}{3}$
C) $\frac{\pi}{3}$
D) $\frac{\pi}{5}$
E) $\frac{7\pi}{3}$
F) $\frac{3\pi}{3}$
G) $\frac{3\pi}{2}$
H) $\frac{3\pi}{4}$
I) $\frac{3\pi}{3}$
J) $\frac{5\pi}{3}$
3) Consider the solid obtained by rotating about the x-axis the region in the first quadrant enclosed by \( y = x^2 + 2 \), \( y = 6 \), and \( x = 0 \). Write out the integral you would use to find the volume of this solid using the shell method.

A) \( \pi \int_{0}^{2} (x^2 + 2)^2 \, dx \)
B) \( 2\pi \int_{0}^{2} x \, (x^2 + 2) \, dx \)
C) \( \pi \int_{0}^{6} y \sqrt{y - 2} \, dy \)
D) \( 2\pi \int_{2}^{6} y \sqrt{y - 2} \, dy \)
E) \( \pi \int_{2}^{6} (x^2 + 2)^2 \, dx \)
F) \( 2\pi \int_{0}^{2} x \, (x^2 + 2) \, dx \)
G) \( \pi \int_{2}^{6} \sqrt{y - 2} \, dy \)
H) \( 2\pi \int_{0}^{6} y \sqrt{y - 2} \, dy \)
I) \( \pi \int_{0}^{6} x \, (x^2 + 2) \, dx \)
J) \( 2\pi \int_{0}^{6} \sqrt{y - 2} \, dy \)

4) A tank in the shape of a cylinder with radius 2 ft and height 6 ft is full of a liquid weighing 80 lb/ft³. Find the work in ft-lb needed to pump one-third of the liquid out of the top of the tank.

A) 120 \( \pi \)
B) 230 \( \pi \)
C) 345 \( \pi \)
D) 460 \( \pi \)
E) 520 \( \pi \)
F) 640 \( \pi \)
G) 730 \( \pi \)
H) 840 \( \pi \)
I) 890 \( \pi \)
J) 940 \( \pi \)
5) It takes 1800 ft-lb of work to stretch a spring 3 ft from its natural length. How many feet beyond its natural length will a force of 2000 lb stretch the spring?

A) 1  
B) 2  
C) 3  
D) 4  
E) 5  
F) 6  
G) 7  
H) 8  
I) 9  
J) 10

6) A motor at the top of an elevator shaft is about to lift a 200 lb weight from the ground, using a uniform cable 180 ft long, the whole cable weighing 40 lb. How much work in ft-lb does the motor do in lifting the weight and the entire cable to the top of the shaft, which is 180 ft above the ground?

A) 9400  
B) 18500  
C) 24800  
D) 32400  
E) 39600  
F) 42200  
G) 46400  
H) 48600  
I) 52400  
J) 56800
7) Evaluate \( \int_0^1 t e^{\pi t} \, dt \).

A) \( \pi e^\pi \)
B) \( \pi e^\pi - 1 \)
C) \( \pi e^\pi - \pi + 1 \)
D) \( \frac{\pi e^\pi + 1}{\pi} \)
E) \( \pi e^\pi + \pi^2 + 1 \)
F) \( \pi^2 e^\pi - e^\pi + 1 \)
G) \( 2\pi e^\pi - e^{\pi^2} - 1 \)
H) \( \frac{e^\pi - e^{\pi^2} + 1}{\pi} \)
I) \( \frac{\pi e^\pi - e^{\pi^2} + 1}{\pi^2} \)
J) \( \frac{\pi e^\pi - e^{\pi^2} + 1}{\pi^2} \)

8) Suppose \( f(x) \) is a function and \( f'(x) \) is continuous. If we are given that \( f(1) = 0 \), \( f(2) = 1 \), and \( \int_1^2 f(x) \, dx = -2 \), then use integration by parts to find \( \int_1^2 x \cdot f'(x) \, dx \). (Hint: \( u = x \) and \( dv = f'(x) \, dx \).)

A) 1
B) 2
C) 3
D) 4
E) 5
F) 6
G) 7
H) 8
I) 9
J) 10
9) Suppose we wanted to approximate the integral \( \int_1^2 \frac{1}{x} \, dx \) with the the Midpoint Rule. Use the Error bound formula for \( M_N \), to find out how large \( N \) should be in order to make sure that the error of our approximation is at most \( 10^{-6} \)?

(Recall: Error(M_N) \( \leq \frac{K_5(b-a)^3}{24N^2} \).

A) 20  
B) 463  
C) 49  
D) 289  
E) 71  
F) 184  
G) 89  
H) 275  
I) 95  
J) 179

10) Solve the integral \( \int_0^\pi \sin^3(x) \cos^4(x) \, dx \).

A) \( \frac{3}{17} \)  
B) \( \frac{3}{27} \)  
C) \( \frac{2}{33} \)  
D) \( \frac{2}{15} \)  
E) \( \frac{1}{18} \)  
F) \( \frac{4}{26} \)  
G) \( \frac{4}{35} \)  
H) \( \frac{4}{25} \)  
I) \( \frac{3}{28} \)  
J) \( \frac{3}{21} \)
11) Using the formula \( \int \tan(x) \, dx = \ln|\sec(x)| + C \), and a trigonometric identity, evaluate \( \int_0^{\frac{\pi}{4}} \tan^3(x) \, dx \).

A) \(1 + \ln(2)\)
B) \(1 - \ln(2)\)
C) \(2 + \ln(2)\)
D) \(2 - \ln(2)\)
E) \((1 + \ln(2))/2\)
F) \((1 - \ln(2))/2\)
G) \((2 + \ln(2))/4\)
H) \((2 - \ln(2))/4\)
I) \(2 + 2\ln(2)\)
J) \(2 - 2\ln(2)\)

12) Using the trigonometric substitution \( t = 2 \sin(\theta) \) solve the integral \( \int_0^{\sqrt{2}} \frac{dt}{(4 - t^2)^{3/2}} \). (recall: \( \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} \))

A) \(1\)
B) \(\frac{1}{3}\)
C) \(\frac{1}{4}\)
D) \(\frac{1}{1}\)
E) \(\frac{\sqrt{3}}{2}\)
F) \(\frac{\sqrt{3}}{2}\)
G) \(\frac{\sqrt{3}}{3}\)
H) \(2\)
I) \(\frac{3}{2}\)
J) \(3\)
13) Using the trigonometric substitution $x = 3\tan(\theta)$, the integral $\int \frac{dx}{\sqrt{x^2+9}}$ will become:

A) $\int \sec(\theta) d\theta$
B) $\int 3\sec(\theta) d\theta$
C) $\int 9\sec(\theta) d\theta$
D) $\int \tan(\theta) d\theta$
E) $\int 3\tan(\theta) d\theta$
F) $\int 9\tan(\theta) d\theta$
G) $\int \sec(\theta)\tan(\theta) d\theta$
H) $\int 3\sec(\theta)\tan(\theta) d\theta$
I) $\int 9\sec(\theta)\tan(\theta) d\theta$
J) $\int \sec^2(\theta) d\theta$

14) Use partial fractions to evaluate $\int \frac{x^2+2}{x^4+x} \, dx \ (x > 0)$.

A) $\ln(2x + 1) + C$
B) $\ln(x^2 + x) + C$
C) $x\arctan(x) + C$
D) $\frac{1}{\arctan(x)} + C$
E) $(2x + 1)^2 + C$
F) $\ln\left(\frac{x^2+1}{x}\right) + C$
G) $\ln\left(\frac{x^2}{x+1}\right) + C$
H) $\ln\left(\frac{2x}{x+1}\right) + C$
I) $\ln(\arctan(x)) + C$
J) $\arctan(\ln(x)) + C$
15) Evaluate the integral \( \int_1^2 \frac{dx}{x(x^2 + 1)} \).

A) \( \ln\left( \frac{2\sqrt{2}}{\sqrt{5}} \right) \)
B) \( \ln\left( \frac{5}{\sqrt{2}} \right) \)
C) \( \ln\left( \frac{3}{\sqrt{8}} \right) \)
D) \( \ln\left( \frac{3}{\sqrt{2}} \right) \)
E) \( \ln\left( \frac{1\sqrt{5}}{\sqrt{3}} \right) \)
F) \( \ln\left( \frac{2}{\sqrt{5}} \right) \)
G) \( \ln\left( \frac{5}{\sqrt{5}} \right) \)
H) \( \ln\left( \frac{7}{\sqrt{2}} \right) \)
I) \( \ln\left( \frac{2}{\sqrt{7}} \right) \)
J) \( \ln(3) \)

16) Evaluate the improper integral \( \int_0^1 x \ln(x) \, dx \), if it is convergent.
If not state that it is divergent.

A) \( \frac{1}{8} \)
B) \( \frac{1}{2} \)
C) 1
D) \( -\frac{1}{2} \)
E) \( -\frac{1}{1} \)
F) 2
G) \( -\frac{1}{4} \)
H) 4
I) \( -\frac{4}{1} \)
J) divergent
PART II: CLEARLY WRITE YOUR SOLUTION AND HOW YOU GOT IT.

Name: ___________________________  ID #: ___________________________

This question will be evaluated not only for having the correct solution but also for clarity. Points may be taken off for confusing and disorganized writing, even when the answer is correct.

17) Given \( \int \frac{x^3}{\sqrt{9-x^2}} \, dx \) \((0 < x < 3)\).

a) Use the trig. substitution \( x = 3 \sin(\theta) \) to rewrite \( \int \frac{x^3}{\sqrt{9-x^2}} \, dx \) as \( \int F(\theta) \, d\theta \).

b) Now use a trigonometric identity to solve \( \int F(\theta) \, d\theta \), in terms of \( \theta \).

c) From (a) & (b), write down the solution of \( \int \frac{x^3}{\sqrt{9-x^2}} \, dx \), in terms of \( x \).
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18) Given $\int 2t \arcsin (t^2) \, dt$.

a) Use the method of integration by parts to write out formulas for $u$ and $dv$ such that you can rewrite $\int 2t \arcsin (t^2) \, dt$ as $u \cdot v - \int v \, du$ (no substitution).

b) Now use the method of substitution to solve the integral $\int v \, du$ from part (a).

c) Using (a) and (b), write out your complete solution for $\int 2t \arcsin (t^2) \, dt$. 