This exam contains sixteen questions. The first fourteen are multiple choice questions and count for five points each. There is no partial credit on these questions, so read each question carefully, check your arithmetic and make sure that you have marked the answer you intended to mark. The last two questions, which are each worth fifteen points, require written answers, and some partial credit might be given. However, no credit will be given for information that is not germane to the problem at hand. Please make sure to write your name and student ID number on the pages that include your answers to the last two questions. In fact, you will get one point on each of these two questions for writing your name and ID number legibly.
1. Compute

\[ \int \frac{dx}{x \ln x} \]

(a) \( \cos(e^x) + C \)
(b) \( e^{\cos x} + C \)
(c) \( x \sin(e^x) + C \)
(d) \( -\ln |x| + C \)
(e) \( \ln |x| + C \)
(f) \( \frac{1}{\ln(x)} + C \)
(g) \( \ln |\ln x| + C \)
(h) \( \frac{1}{x} + C \)
2. Compute \( \frac{dy}{dx} \) when
\[
y = \int_0^{x^2} \cos \sqrt{t} \, dt
\]

(a) \( \cos \sqrt{x} \)
(b) \( \cos|x| \)
(c) \( \cos|x| - 1 \)
(d) \( x^2\cos \sqrt{t} \)
(e) \( x^2\cos \sqrt{x} \)
(f) \( 2x\cos|x| \)
(g) \( \cos x + C \)
(h) \( \cos|x| - 1 \)
3. Compute

\[ \int_0^\pi x \sin\left(\frac{x}{2}\right) \, dx \]

(a) 0  
(b) 0.5  
(c) 2  
(d) 4  
(e) \pi  
(f) \frac{\pi}{2}  
(g) \frac{\pi}{4}  
(h) \pi - 1
4. Expand $\frac{x+4}{(x+1)^2}$ by partial fractions.

(a) $\frac{1}{x+1} + \frac{x+3}{(x+1)^2}$
(b) $\frac{2}{x+1} + \frac{x+2}{(x+1)^2}$
(c) $\frac{3}{x+1} + \frac{x+1}{(x+1)^2}$
(d) $\frac{4}{x+1} + \frac{x}{(x+1)^2}$
(e) $\frac{1}{x+1} + \frac{3}{(x+1)^2}$
(f) $\frac{3}{x+1}$
(g) $\frac{3}{(x+1)^2}$
(h) impossible since undefined at $x = -1$
5. Evaluate

\[ \int_{2}^{\infty} \frac{1}{\ln(x)} \, dx \]

(a) Converges, by direct comparison with comparison function \( \frac{1}{x^2} \).
(b) Diverges, by direct comparison with the comparison function \( \frac{1}{x^2} \).
(c) Converges, by direct comparison with the comparison function \( e^x \).
(d) Diverges, by direct comparison with the comparison function \( e^x \).
(e) Converges, by direct comparison with the comparison function \( \frac{1}{x} \).
(f) Diverges, by direct comparison with the comparison function \( \frac{1}{x} \).
(g) Converges, by direct comparison with the comparison function \( e^{-x} \).
(h) Diverges, by direct comparison with the comparison function \( e^{-x} \).
6. The region between the curve \( y = 2\sqrt{x} \), \( 0 \leq x \leq 1 \), and the \( x \)-axis is revolved about the \( x \)-axis to generate a solid. Compute the volume of this solid.

(a) \( \pi/2 \)
(b) \( \pi \)
(c) \( 2\pi \)
(d) \( 5\pi/2 \)
(e) \( 5\pi \)
(f) \( \pi^2 \)
(g) \( \pi^2/2 \)
(h) \( 16\pi/15 \)
7. Suppose \[
\frac{dy}{dx} = \frac{1}{y}
\]
and \(y(0) = 1\). Find \(y(4)\).

(a) 0  
(b) ±2  
(c) ±3  
(d) ±4  
(e) \(e^4\)  
(f) \(\ln 4\)  
(g) 1/4  
(h) –1/4
8. Compute the sum of the series

\[ \sum_{n=1}^{\infty} \left( \cos \left( \frac{1}{n} \right) - \cos \left( \frac{1}{n+1} \right) \right) \]

(a) diverges to \( \infty \)
(b) diverges to \(-\infty\)
(c) diverges but not to \(-\infty\) or \(\infty\)
(d) \(-1\)
(e) \(-1 + \cos 1\)
(f) \(0\)
(g) \(1\)
(h) \(\cos 1\)
9. Which of the following three series is convergent?

\[ (A) \sum_{n=1}^{\infty} \frac{n + 2}{n^{3/2}} \quad (B) \sum_{n=1}^{\infty} \frac{3}{n^4 - 2n^2} \quad (C) \sum_{n=1}^{\infty} \frac{n}{\sqrt{n^4 + 5}} \]

(a) A only  
(b) B only  
(c) C only  
(d) A and B only  
(e) A and C only  
(f) B and C only  
(g) all  
(h) none
10. If the Maclaurin series for $xe^{x^2}$ is $\sum c_n x^n$, find $c_7$.

(a) 0
(b) 1
(c) 1/2
(d) 1/6
(e) 1/7
(f) 1/24
(g) 1/7!
(h) 1/8!
11. Estimate \( \int_0^1 \sin x^2 \, dx \) with an error less than 0.001.

(a) \( \frac{13}{42} \)
(b) \( \frac{1}{3} \)
(c) \( \frac{1}{42} \)
(d) \( \frac{1}{1320} \)
(e) \( \frac{1}{7} \)
(f) \( \frac{2}{7} \)
(g) \( \frac{1}{24} \)
(h) \( \frac{5}{24} \)
12. Compute $e^{-i\pi/2}$

(a) 0  
(b) 1  
(c) $-1$  
(d) $i$  
(e) $-i$  
(f) $e$  
(g) $-e$  
(h) $1 + i$
13. If the Binomial Series for the function

\[ f(x) = (1 + x^2)^{4/3} \]

is given by \( \sum c_n x^n \), find \( c_4 \).

(a) \(-1\)  
(b) 0  
(c) 1  
(d) 1/2  
(e) 3/4  
(f) 2/7  
(g) 4/3  
(h) 2/9
14. Find the area inside one leaf of the four-leaved rose

\[ r = \cos(2\theta) \]

(a) \( \pi/8 \)
(b) \( \pi/4 \)
(c) \( \pi/3 \)
(d) \( \pi/2 \)
(e) \( \pi \)
(f) 0.5
(g) 1.5
(h) 2.5
15. Find the Taylor Series generated by \( f(x) = \sin x \) at \( a = \pi/2 \). Show your work. Be sure to include the general term of the series.
16. Identify the symmetries of the curve

\[ r = 1 + \cos \theta \]

and sketch this curve. Show all work.