1. Find the general solution of \( y'' - y' - 2y = 0 \).

\[ (\lambda - 2)^2 = 0 \quad \Rightarrow \quad \lambda_1 = \lambda_2 = 2 \]

Therefore, the general solution is:

\[ y(t) = Ae^{2t} + Be^{2t} \]

\[ y(t) = A e^{2t} + B e^{2t} \]
2. Find the general solution of \( y'' - 2y' = 0 \).

\[ r^2 - 2r = 0 \]
\[ r = 0, \quad r = 2 \]

G.S. \( y(t) = Ae^{0t} + Be^{2t} \)

\[ \frac{y(t) = A + Be^{2t}}{y(t) = A + Be^{2t}} \]
3. Each of the following function pairs consists of two solutions to the equation \( y'' - y' = 0 \). Which of the pairs constitute a fundamental system of solutions?

I) \( 1, e^t \)  
II) \( 0, e^t \)  
III) \( e^t, 2e^t \)

(A) I  
(B) II  
(C) III  
(D) I, II  
(E) I, III  
(F) II, III  
(G) I, II, III  
(H) None of them

\[ w(1, e^t) = \begin{vmatrix} 1 & e^t \\ 0 & e^t \end{vmatrix} = e^t \neq 0 \; \checkmark \]

\[ w(0, e^t) = \begin{vmatrix} 0 & e^t \\ 0 & e^t \end{vmatrix} = 0 \; \times \]

\[ w(e^t, 2e^t) = \begin{vmatrix} e^t & 2e^t \\ e^t & 2e^t \end{vmatrix} = 0 \; \times \]
4. Find the general solution of \( y''' + 4y' + 5y = 0 \).

A) \( Ae^t + Be^{-2t} \)

B) \( A + Be^{2t} \)

C) \( Ae^{-2t} + Bte^{-2t} \)

D) \( Ae^{-t^2} + Bte^{-t^2} \)

E) \( Acos t + Bsin t \)

F) \( Acos t + Btcos t \)

G) \( Ae^{-2t}cos t + Be^{-2t}sin t \)

H) \( Ae^{t}cos t + Be^{t}sin t \)

I) \( A + Bt \)

J) None of the above

\[
\begin{align*}
& r^2 + 4r + 5 = 0 \\
& r = -2 \pm i \\
& \lambda = -2, \mu = 1 \\
& y = Ae^{-2t}cos t + Be^{-2t}sin t
\end{align*}
\]
5. Solve the initial value problem  \( y'' + 4y = 0, \ y(0) = 1, \ y'(0) = 0. \)

A) \( \sin 2t \)

B) \( \cos 2t \)

C) \( e^{3t} \)

D) \( t \ e^{3t} \)

E) \( \ln(t) \)

F) \( t \ln(t) \)

G) \( -e^{2t} \)

H) \( 3t \cos 2t \)

I) \( -(3/4) \ t \cos 2t \)

J) None of the above

\[ r^2 + 4 = 0 \]
\[ \Rightarrow r = \pm 2i \]
\[ \lambda = 0, \ \mu = 2 \ so \]

\[ y(t) = A \cos 2t + B \sin 2t \]

\[ y'(t) = -2A \sin 2t + 2B \cos 2t \]

\[ 1 = y(0) = A \cos 0 + B \sin 0 = A \]

\[ 0 = y'(0) = -2A \sin 0 + 2B \cos 0 = 2B \]

\[ A = 1, \ B = 0 \]

\[ y(t) = \cos 2t \]
6. Find the general solution of \( y'' + y' + \frac{1}{4} y = 0 \).

A) \( Ae^t + Be^{2t} \)

B) \( A + Be^{2t} \)

C) \( Ae^{-2t} + Be^{2t} \)

D) \( Ae^{-t/2} + B e^{t/2} \)

E) \( Acos \ t + Bsin \ t \)

F) \( Acos \ t + Bcos \ t \)

G) \( Ae^{-2t} \cos \ t + Be^{2t} \sin \ t \)

H) \( Ae^t \cos \ t + Be^t \sin \ t \)

I) \( A + Bt \)

J) None of the above

\[ \xi_{\text{char}} \quad r^2 + r + \frac{1}{4} = 0 \]

\[ (r + \frac{1}{2})^2 = 0 \]

\[ r = -\frac{1}{2}, -\frac{1}{2} \]

\[ y(t) = A e^{-\frac{1}{2}t} + B t e^{-\frac{1}{2}t} \]
7. Solve the initial value problem $y'' - 10y' + 25y = 0$, $y(0) = 0$, $y'(0) = 1$.

A) $\sin 2t$

B) $\cos 2t$

C) $e^{5t}$

D) $t \, e^{5t}$

E) $\ln(t)$

F) $t \ln(t)$

G) $-e^{2t}$

H) $3t \cos 2t$

I) $-(3/4) \, t \cos 2t$

J) None of the above

$x_{char} \quad r^2 - 10r + 25 = 0$

$r = 5, 5$.

$y(t) = Ae^{5t} + Bte^{5t}$

$y'(t) = 5Ae^{5t} + B(e^{5t} + 5te^{5t})$

$0 = y(0) = Ae^0 + B \cdot 0 \cdot e^0 = A$

$1 = y'(0) = 5Ae^0 + B(1 + 0) = 5A + B$

$5A + B = 1 \quad \Rightarrow B = 1$

$y(t) = t e^{5t}$
8. Use the method of undetermined coefficients to find a particular solution $y_p$ to the equation $y'''' - 2y' - 3y = 3e^{2t}$.

A) $\sin 2t$

B) $\cos 2t$

C) $e^{2t}$

D) $te^{2t}$

E) $\ln(t)$

F) $t \ln(t)$

G) $e^{2t} \left(-e^{2t}\right)$

H) $3t \cos 2t$

I) $-(3/4) t \cos 2t$

J) None of the above

\[ y'''' - 2y' - 3y = \mathbf{0} \]

\[ \lambda^4 - 2\lambda - 3 = 0 \]

\[ \lambda = -1, \ 3 \]

G.S. or $\mathbf{w}_n$ is $C_1 e^{-t} + C_2 e^{3t}$

$g(t) = 3e^{2t}$

Thus $y_p = A e^{2t}$

OK (does not solve reduced)

$y_p' = 2A e^{2t}, \ y_p'' = 4A e^{2t}$

$y_p''' = 8A e^{2t}, \ y_p'''' = 16A e^{2t}$

\[ y_p'''' = 16A e^{2t} - 4(2A e^{2t}) - 3A e^{2t} = 3e^{2t} \]

\[ 4A - 4A - 3A = 3 \]

\[ \Rightarrow A = -1 \]

\[ y_p = -e^{2t} \]
9. Use the method of undetermined coefficients to find a particular solution $y_p$ to the equation $y'' + 4y = 3 \sin 2t$.

A) $\sin 2t$

B) $\cos 2t$

C) $e^{3t}$

D) $te^{3t}$

E) $\ln(t)$

F) $t \ln(t)$

G) $-e^{2t}$

H) $3t \cos 2t$

I) $-(3/4)t \cos 2t$

J) None of the above
10. Which of the following cannot be solved using the method of undetermined coefficients?

I) $y'' + y = \tan t$  
II) $t^2y + y = \cos t$  
III) $y'' + 3y' - 6y = e^{3t}$

A) I
B) II
C) III
D) I, II
E) I, III
F) II, III
G) I, II, III
H) None of them

I. N.G. Tan t has no many domin. types
II. N.G. Has non-constant cor. ff.
III. OK
11. Solve the initial value problem $y'' = t^2$, $y(1) = 0$, $y'(1) = 1$ (assume that $t > 0$).

A) $\sin 2t$

B) $\cos 2t$

C) $e^{5t}$

D) $te^{5t}$

E) $\ln(t)$

F) $t \ln(t)$

G) $-e^{2t}$

H) $3t \cos 2t$

I) $-(3/4) t \cos 2t$

J) None of the above

\[ y''(t) = \frac{t}{t} \quad t > 0 \]

So $y'(t) = \int \frac{t}{t} \, dt = \ln t + c$

But $1 = y'(1) = \ln 1 + c = c$\quad \Rightarrow \quad y'(t) = \ln t + 1$

\[ y(t) = \int y'(t) \, dt = \int (\ln t + 1) \, dt \]

\[ = (t \ln t - t) + t + D \]

\[ y(t) = t \ln t + D \]

Hence:\n
\[ y(1) = 1 \ln 1 + D = D \]

\[ \therefore \quad y(t) = t \ln t \]

or use $u, v$. 
12. A mass of 2 Kg is suspended at equilibrium from a spring whose Hooke’s constant is \( k = 10 \). It is then pushed downwards at 1 m/sec. There is no friction. How many seconds after the push does the mass \textit{first} return to the equilibrium position?

A) \( \pi \)

B) \( \frac{3\pi}{4} \)

C) \( \frac{\pi}{2} \)

D) \( 2\pi \)

E) \( 5\pi \)

F) \( \frac{\pi}{5} \)

G) \( \frac{\pi}{3} \)

H) \( \frac{\pi}{\sqrt{5}} \)

I) \( \frac{\pi}{\sqrt{3}} \)

J) None of the above

The period \( T \) of the \textit{undamped oscillation} \textit{is}

\[
T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{2}{10}}
\]

\[
T = \frac{2\pi}{\sqrt{5}}
\]

The answer to the problem is \( \frac{1}{2} \pi \) \text{ or } \( \frac{\pi}{\sqrt{5}} \).
13. A mass of 5 Kg is suspended from a spring of Hooke's constant \( k = 5 \) in a viscous medium which exerts a force \( 10|v| \) on a moving body. (i.e. \( c = 10 \)). Beginning at time \( t = 0 \) a force of \( F(t) = \sin t \) N is applied. There will be a transient part of the solution, which vanishes as \( t \) increases, and an ongoing steady state solution. Disregard the transient, and find the steady state motion of the mass.

A) \( \sin 2t \)  
B) \( \cos 2t \)  
C) \( 10 \cos t \)  
D) \( 10 \sin t \)  
E) \( -10 \cos t \)  
F) \( -(1/10) \cos t \)  
G) \( (1/10) \sin t \)  
H) \( t \cos 2t \)  
I) \( -t \cos 2t \)  
J) None of the above

\[
\text{Let } \chi(t) = \text{DISPLACEMENT FROM NEUTRAL.} \\
\]
\[
\text{CON IS:} \\
\]
\[
5 \chi'' + 10 \chi' + 5 \chi = \sin t \\
\]
\[
\text{G, SOLN OF } \chi_R \text{ IS} \\
C_1 e^{-t} + C_2 t e^{-t} \\
\]
\[
\text{USE } C_1, C_2 \text{ THAT } \chi_R = A \sin t + B \cos t \\
\]
\[
\Rightarrow \chi_R = -\frac{1}{10} \cos t \\
\]
\[
\text{G.S. } \chi(t) = C_1 e^{-t} + C_2 t e^{-t} - \frac{1}{10} \cos t \\
\text{TRANSITION } \underline{SS} \\
\]
\[
\tilde{SS. } \text{ SOLN IS } \chi_P(t) = -\frac{1}{10} \cos t \\
\]
14. Suppose the function \( y(x) = \sum_{n=0}^{\infty} a_n x^n \) is a series solution to the equation \( y'' + xy' + 2y = 0 \). Find the recursion relation for the coefficients \( a_n \).

A) \( a_{n+1} = \frac{1}{n+1} a_n \)

B) \( a_{n+1} = \frac{-1}{n+2} a_n \)

C) \( a_{n+1} = \frac{n-1}{(n+2)(n+3)} a_n \)

D) \( a_{n+2} = \frac{1}{n+1} a_n \)

E) \( a_{n+2} = \frac{-1}{n+1} a_n \)

F) \( a_{n+2} = \frac{1}{(n+1)(n+2)} a_n \)

G) \( a_{n+2} = \frac{-1}{(n+1)(n+2)} a_n \)

H) \( a_{n+2} = \frac{2n-1}{n+2} a_n \)

I) \( a_{n+2} = \frac{n-1}{(2n+2)(n+1)} a_n \)

J) None of the above
15. Which of the following numbers are possible as the radius of convergence of a power series solution \( y = \sum_{n=0}^{\infty} a_n x^n \) of the equation \((x^2 + 1)y'' + 4xy' + 4y = 0\)?

I) \(1/4\)  II) \(1/3\)  III) \(1/2\)

A) I  
B) II  
C) III  
D) I, II  
E) I, III  
F) II, III  
G) I, II, III  
H) None of them

\[ p(x) = x^2 + 1 \]

Roots in CPX plane \( \begin{align*}
&+\sqrt{3}i \\
&-\sqrt{3}i
\end{align*} \)

Roots: \( e^{i\theta} \)

\[ p(1) \text{ at } (0, 0) \text{ is } 5 < 2 \Rightarrow \text{NE} \]

\[ \therefore \text{R. of C. of soln } \geq 1 \]

\[ \text{NONE ARE POSSIBLE} \]
Math 217 exam 2 hand-graded problem.

In an environment with no friction, a 1 kg mass is suspended at equilibrium from a spring whose Hooke's constant is \( k = 4 \). At \( t = 0 \) it is pushed downward with a speed of 1 m/sec. Let \( x(t) \) = displacement from neutral position at time \( t \).

a) Find an expression for \( x(t) \).

b) What is the frequency \( \omega_0 \) of the motion?

c) Suppose everything remains the same, except that the motion takes place in a viscous medium whose frictional constant is \( c = 2 \). Now find \( x(t) \).

d) For the motion of part c), what is the Quasi-frequency \( \omega_d \)?

\( a) \quad x'' + 4x = 0, \quad x(0) = 0, \quad x'(0) = 1 \)

So
This is \( x(t) = A \cos 2t + B \sin 2t \)

\[ x'(t) = -2A \sin 2t + 2B \cos 2t \]

\[ 1 = x'(0) = 2B \cdot 1 \Rightarrow B = \frac{1}{2} \]

\[ x(t) = \frac{1}{2} \sin 2t \]

\( b) \quad \omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{4}{1}} = 2 \quad \square \quad \omega_0 = 2 \]

\( c) \quad \text{Now } x(t) \text{ satisfies the ivp:} \)

\( x'' + 2x' + 4x = 0, \quad x(0) = 0, \quad x'(0) = 1 \)

\( r^2 + 2r + 4 = 0 \Rightarrow r = -1 \pm \sqrt{3}i \)

\[ \lambda = -1, \mu = \sqrt{3} \]

\( G, S. I S \quad x(t) = A e^{-t} \cos \sqrt{3}t + B e^{-t} \sin \sqrt{3}t \)

\[ A(1 \cos \sqrt{3}t - \sin \sqrt{3}t) + B(\sin \sqrt{3}t - \cos \sqrt{3}t) \]
We know \( x(0) = 0 \) so
\[
o = x(0) = Ae^{0 \cdot 0} + Be^{0 \sin 0} = A
\]

\[
\therefore \quad x(t) = Be^{-t \sin \sqrt{3} t}
\]

\[
x'(t) = B \left[ -e^{-t \sin \sqrt{3} t} \sqrt{3} \cos \sqrt{3} t \right]
\]

\[
1 = x'(0) = B \left[ e^{0 \sin 0 + \sqrt{3} \cdot 0} \right]
\]

\[
1 = B \sqrt{3}
\]

\[
\underbrace{B = \frac{1}{\sqrt{3}}} \Rightarrow \quad x(t) = \frac{1}{\sqrt{3}} e^{-t \sin \sqrt{3} t}
\]

d) We know \( \omega d = \frac{\sqrt{4 \cdot h m - c^2}}{2m} \)

Here, \( \omega d = \frac{\sqrt{4 \cdot 4 \cdot 1 - 2^2}}{2 \cdot 1} = \frac{\sqrt{12}}{2} = \sqrt{3} \)

\[
\omega d = \sqrt{3}
\]

Note: This could have been "read off" the F.C.N. \( \sin \sqrt{3} t \) in \( x(t) \) above.