1. \( \frac{dy}{dx} = \sin(x) \cos(y) \), with \( y(\pi/4) = 1 \). Find the approximate value of the \( y \)-intercept of the tangent line to the solution curve at \( x = \pi/4 \).

(A) 0.01
(B) 0.11
(C) 0.21
(D) 0.31
(E) 0.41
(F) 0.51
(G) 0.61
(H) 0.71
(I) 0.81
(J) 0.91
2. $y'' - 2y' + y = 0$, where the derivative is taken with respect to $t$. Let $p = y'$ and let $X = \begin{pmatrix} y \\ p \end{pmatrix}$. Then $\frac{dX}{dt} = AX$ for some 2-by-2 matrix $A$. What is the determinant of $A$?

(A) 9
(B) 8
(C) 7
(D) 6
(E) 5
(F) 4
(G) 3
(H) 2
(I) 1
(J) 0
\[ \frac{dx}{dt} = x + 1 \]

3. with \( x(0) = 0, y(0) = -1 \). What is the type of the solution curve?

\[ \frac{dy}{dt} = y + 2 \]

\begin{align*}
(A) \text{ a circle} \\
(B) \text{ a hyperbola} \\
(C) \text{ a line} \\
(D) \text{ a parabola} \\
(E) \text{ a hyperbolic cosine curve} \\
(F) \text{ a hyperbolic sine curve} \\
(G) \text{ a figure 8} \\
(H) \text{ a cardioid} \\
(I) \text{ a four-leaved rose} \\
(J) \text{ undecidable}
\end{align*}
4. $\frac{dy}{dt} = 2y - 3t, y(0) = 1$. Use Euler's method to find the approximate value of $y(0.1)$ with step size 0.05.

(A) 0.0110
(B) -1.4142
(C) -2.2360
(D) 2.7182
(E) 3.1416
(F) 1.2025
(G) 4.4310
(H) 5.1968
(I) -5.1968
(J) 6.0012
5. \( y'' + 4y = t \sin(2t), y(0) = y'(0) = 0. \) Let \( F(s) \) be the Laplace transform of \( y \). What is \( F(1) \)?

(A) 0.032  
(B) 0.056  
(C) 0.100  
(D) 0.280  
(E) 1.012  
(F) -1  
(G) -2  
(H) -3  
(I) -4  
(J) -5
6. If \((xy^2 + bzx^2)dx + (x + y)x^2dy = 0\) is exact, then what is \(b\) equal to?

(A) 0
(B) 2
(C) 4
(D) 6
(E) 8
(F) 9
(G) 7
(H) 5
(I) 3
(J) 1
7. \( ydx + (2xy - e^{-2x})dy = 0, \ y(0) = 1 \). What is \( x \) approximately equal to when \( y \) is \( e \)?

(A) 0.000123
(B) 0.020000
(C) 0.009071
(D) 0.010480
(E) 0.007832
(F) 0.004354
(G) 0.007548
(H) 0.123567
(I) 0.100000
(J) 0.123450
8. \( y'''' + 4y' + 5y = 0 \). Its characteristic equation has one real root \( a \) and two conjugate complex roots \( b \pm \imath c, c > 0 \), where \( \imath = \sqrt{-1} \). What is \( a^2 + b^2 + c^2 ? \)

(A) 2
(B) 4
(C) 6
(D) 8
(E) 10
(F) 16
(G) 18
(H) 20
(I) 25
(J) 36
9. $ty' + y = \sin(t), t > 0$. If $y(\pi/2) = 0$, what is $y(\pi)$ approximately equal to?

(A) 0.50
(B) 0.32
(C) 0.14
(D) 0.99
(E) 0.82
(F) 0.75
(G) 1.00
(H) 1.24
(I) 1.41
(J) 1.73
10. \( y'' + y' - 2y = 2t \). We know \( e^t \) and \( e^{-2t} \) are two fundamental solutions. Suppose the general solution is of the form \( c_1 e^t + c_2 e^{-2t} + P(t) \), where \( c_1, c_2 \) are constants and \( P(t) \) is a polynomial. What is \( P(3/2) \)?

(A) 2  
(B) 1  
(C) 0  
(D) -1  
(E) -2  
(F) -3  
(G) 4  
(H) -4  
(I) 5  
(J) 6
11. \( x(1-x^2)^3y''+(1-x^2)^2y'+2(1+x)y = 0 \). Classify the singular points 
-1, 0, 1, in the sequencial order, as regular singular (RS) or irregular 
singular (IS)?

(A) RS, RS, RS
(B) RS, RS, IS
(C) RS, IS, RS
(D) RS, IS, IS
(E) IS, RS, RS
(F) IS, RS, IS
(G) IS, IS, RS
(H) IS, IS, IS
(I) None of the above
(J) Undecidable
12. $y'' + x^2 y = 0$. Let $y = \sum_{n=0}^{\infty} a_n x^n$ be a series solution to the equation with the initial conditions $y(0) = 1, y'(0) = -1$. What is $30(a_0 + a_1 + a_2 + a_3 + a_4 + a_5)$ equal to?

(A) 0
(B) 1
(C) 2
(D) 3
(E) -3
(F) -2
(G) -1
(H) 4
(I) 6
(J) -4
13. $A = \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix}$. Let $e^{At} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$. What is $\beta/\gamma$?

(A) 1  
(B) -1  
(C) 2  
(D) -2  
(E) 4  
(F) -4  
(G) 1/2  
(H) -1/2  
(I) 1/4  
(J) -1/4
14. Consider the system \[
\frac{dx}{dt} = -x + 2xy \\
\frac{dy}{dt} = y - x^2 - y^2
\] It has several critical points. Let \( \alpha \) be the smallest \( x \)-coordinate of all critical points, and let \( \beta \) be the largest \( y \)-coordinate of all the critical points. What is \( \beta - \alpha \)?

(A) 1  
(B) 1.5  
(C) 2  
(D) 2.5  
(E) 3  
(F) 3.5  
(G) 4  
(H) 4.5  
(I) 5  
(J) 5.5
15. \( \frac{dX}{dt} = \begin{pmatrix} 2 & -5/2 \\ 9/5 & -1 \end{pmatrix} X \). What is the type and stability of the critical point of the system?

(A) A center, stable
(B) An improper nodal point, unstable
(C) An improper nodal point, asymptotically stable
(D) A spiral point, unstable
(E) A spiral point, asymptotically stable
(F) A proper nodal point, unstable
(G) A proper nodal point, asymptotically stable
(H) A saddle point, unstable
(I) The system has more than one critical point
(J) Undecidable
16. \( X_1 = \begin{pmatrix} t \\ 1 \end{pmatrix} \) and \( X_2 = \begin{pmatrix} t^2 \\ 2t \end{pmatrix} \). Find the Wronskian \( W(X_1, X_2) \) at \( t = 1 \).

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5
(F) 6
(G) 7
(H) 8
(I) 9
(J) 10
17. $\frac{dX}{dt} = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \text{X}$. Find the slope of the line through the origin to which a generic solution curve is tangent.

(A) 1
(B) 1.5
(C) 2
(D) 2.5
(E) 3
(F) 3.5
(G) 4
(H) 4.5
(I) 5
(J) 5.5
13. \( \frac{dy}{dx} = \frac{2x + 3y}{3x - 2y} \) What is the type of a generic solution curve?

(A) A closed curve
(B) An improper nodal curve
(C) A proper nodal curve
(D) A spiral curve
(E) A ray emanating from the origin
(F) A hyperbola-like curve repelled from the origin
(G) undecidable
(H)
(I)
(J)
19. \( \frac{dX}{dt} = \begin{pmatrix} 1 & t \\ t & t^2 \end{pmatrix} X. \) Let \( X_1(t) \) be the solution with \( X_1(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \)

and let \( X_2(t) \) be the solution with \( X_2(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \). Let \( W(t) \) be the Wronskian \( W(X_1(t), X_2(t)) \). What is \( \ln(W(3)) \) equal to?

(A) 15
(B) 14
(C) 13
(D) 12
(E) 11
(F) 10
(G) 9
(H) 8
(I) 7
(J) 6
20. Consider the triple spring-mass system as depicted in the figure.

Let $v_1 = du_1/dt$, $v_2 = du_2/dt$, $v_3 = du_3/dt$. Let $X = \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{pmatrix}$ Then

$$\frac{dX}{dt} = AX$$

for some 6-by-6 constant matrix $A$. Find the $(4,3)$-entry of $A$.

(A) 1  
(B) -1  
(C) 2  
(D) -2  
(E) 3  
(F) -3  
(G) 4  
(H) -4  
(I) 5  
(J) -5

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$\begin{array}{c}
\text{Equilibrium State} \\
\text{Wall} \\
\text{Frictionless horizontal plane}
\end{array}$

$m_i$: $i$th mass  
$k_i$: $i$th spring constant  
$m_1 = m_2 = m_3 = 1$, $k_1 = 1$, $k_2 = 2$, $k_3 = 3$.  

$U_i$: The displacement of the $i$th mass from its equilibrium state.