Math 233, Fall 2008

Midterm Exam I

September 24, 2008

• Total points = 100.

• Duration = 120 minutes.

• The exam has 15 questions: 13 multiple choice questions and 2 hand-graded questions.

• Each multiple choice question has 6 points.

• For the hand graded questions, show your work to allow for partial credit.

• Good Luck!!
1. If \( P = (2, 3, -1) \) and \( Q = (5, 2, -3) \). Which vector is a unit vector in the direction of \( \overrightarrow{PQ} \)?

(a) \( \langle \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{-2}{\sqrt{14}} \rangle \)

(b) \( \langle \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}} \rangle \)

(c) \( \langle \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}} \rangle \)

(d) \( \langle \frac{3}{\sqrt{26}}, \frac{-1}{\sqrt{26}}, \frac{-4}{\sqrt{26}} \rangle \)

(e) \( \langle \frac{3}{\sqrt{26}}, \frac{-1}{\sqrt{26}}, \frac{4}{\sqrt{26}} \rangle \)

(f) \( \langle \frac{3}{\sqrt{26}}, \frac{1}{\sqrt{26}}, \frac{4}{\sqrt{26}} \rangle \)
2. If \( \mathbf{v} = \langle 2, 3, -1 \rangle \), and \( \mathbf{u} = \langle 1, -3, 7 \rangle \), what is \( \mathbf{v} \times \mathbf{u} \)?

(a) \( \langle 18, 15, -9 \rangle \)

(b) \( \langle 18, -15, -9 \rangle \)

(c) \( \langle 18, 13, -9 \rangle \)

(d) \( \langle 18, -13, -9 \rangle \)

(e) \( \langle -18, -15, 9 \rangle \)

(f) \( \langle -18, -13, 9 \rangle \)
3. If \( \mathbf{v} = <1, 9, -3> \). Which of the following vectors is perpendicular to \( \mathbf{v} \)?

(a) \( < -1, -9, 3 > \)

(b) \( < -1, 9, 3 > \)

(c) \( < -3, 1, 2 > \)

(d) \( < 3, -1, 2 > \)

(e) \( < -3, \frac{1}{3}, 1 > \)
4. What is the center and the radius of the sphere with equation
\[ x^2 + y^2 + z^2 - 2x + 2y + 3z = \frac{-1}{4} \]?

(a) center = (1, -1, 3); radius = 4

(b) center = (1, 1, 3); radius = 2

(c) center = (1, 1, 3); radius = \frac{1}{2}

(d) center = (1, -1, \frac{3}{2}); radius = \frac{1}{2}

(e) center = (1, -1, -\frac{3}{2}); radius = 4

(f) center = (1, -1, -\frac{3}{2}); radius = 2
5. What is the distance from the point $Q(1, -1, 2)$ to the plane with equation $2x + y - z = 5$?

(a) 1

(b) $\frac{1}{\sqrt{6}}$

(c) 6

(d) $\sqrt{6}$

(e) $\frac{1}{6}$
6. What is the area of the triangle with vertices \( P(0, 1, 2) \), \( Q(-1, 2, 2) \), and \( R(4, -1, 0) \)?

(a) \( \sqrt{44} \)

(b) \( \sqrt{3} \)

(c) \( \sqrt{11} \)

(d) \( \sqrt{12} \)

(e) \( \sqrt{48} \)
7. What is the cosine of the angle between the two planes given by equations
\[ 2x + y + z = 4, \] and \[ 3x - y - 3z - 1 = 0? \]

(a) \( \frac{2}{\sqrt{102}} \)

(b) \( \frac{-2}{\sqrt{102}} \)

(c) \( \frac{-4}{\sqrt{102}} \)

(d) \( \frac{4}{\sqrt{102}} \)

(e) \( \frac{2}{\sqrt{114}} \)

(f) \( \frac{-2}{\sqrt{114}} \)
8. What is the equation of a plane passing through the point $P(5, 7, -1)$ and parallel to the plane $x + y + 2z = 1$?

(a) $x + y + 2z = 10$

(b) $x - y + 2z = -4$

(c) $2x + 2z = 6$

(d) $-x + y + z = 1$

(e) $x - y - 3z = 1$
9. Which of the following equations is the vector equation of the line of intersection of the two planes given by $x + y = 2$ and $x - y - 3z = 0$?

(a) $t < -5, 1, -2 > + < 0, 2, \frac{3}{2} >$

(b) $t < 5, -1, 2 > + < 3, 0, 1 >$

(c) $t < -5, 1, -2 > + < 1, 1, 0 >$

(d) $t < -3, -3, -2 > + < 0, 2, \frac{3}{2} >$

(e) $t < -3, 3, -2 > + < 0, 2, 0 >$

(f) $t < 3, -3, 2 > + < 1, 1, 0 >$
10. Assume \( \mathbf{u}(t) \) and \( \mathbf{w}(t) \) are two vector functions such that

\[
\mathbf{u}(t) = \langle t^2, \sin t, 1 + 2t \rangle, \\
\mathbf{w}(0) = \langle 1, 0, -2 \rangle, \\
\mathbf{w}'(0) = \langle 1, 1, 3 \rangle.
\]

If \( f(t) = \mathbf{u}(t) \cdot \mathbf{w}(t) \), what is \( f'(0) \)?

(a) -1

(b) 1

(c) 0

(d) 2

(e) -2
11. What is the parametric equation of the tangent line to the curve traced by \( \mathbf{r}(t) = (\cos t) \mathbf{i} + e^t \mathbf{j} + 2t \mathbf{k} \) at \( t = 0 \)?

(a) \( x = s; \quad y = 1 - s; \quad z = 2 \)

(b) \( x = s; \quad y = 1 + s; \quad z = 2s \)

(c) \( x = 1; \quad y = 1 - s; \quad z = 2s \)

(d) \( x = 1; \quad y = 1 + s; \quad z = 2s \)

(e) \( x = s; \quad y = 1 + s; \quad z = 2 \)
12. The Assume \( \mathbf{r}(t) \) is the position vector of a particle in space. If the acceleration vector is \( \mathbf{a}(t) = (\cos t) \mathbf{i} + (\sin t) \mathbf{j} \), the initial position \( \mathbf{r}(0) = -\mathbf{i} + 5\mathbf{k} \), and the initial velocity vector is \( \mathbf{v}(0) = \mathbf{j} \). What is the position vector at \( t = \frac{\pi}{2} \)?

(a) \(-2\mathbf{i} - (1 + \pi)\mathbf{j} + \mathbf{k}\)

(b) \((-2 + \pi)\mathbf{i} + \mathbf{k}\)

(c) \(-(1 + \pi)\mathbf{j} + 5\mathbf{k}\)

(d) \((2 + \pi)\mathbf{i} + \mathbf{k}\)

(e) \(-\mathbf{i} + \mathbf{j} + 5\mathbf{k}\)

(f) \((\pi - 1)\mathbf{j} + 5\mathbf{k}\)
13. What is the point on the curve

\[ \mathbf{r}(t) = (3 \sin t) \mathbf{i} - (3 \cos t) \mathbf{j} + 4t \mathbf{k} \]

at a distance \( \frac{5\pi}{2} \) units along the curve from the point \((0, -3, 0)\) in the direction of increasing arc length?

(a) \((0, 3, 4\pi)\)

(b) \((3, 0, 2\pi)\)

(c) \((-3, 0, -2\pi)\)

(d) \((0, 3, 2\pi)\)

(e) \((0, -3, -4\pi)\)

(f) \((3, 0, 4\pi)\)
Hand-Graded Question [12 points]

14. Assume that
\[ r(t) = (2t - 1) \mathbf{i} + (t^2 - 1) \mathbf{j} + (4t - 3) \mathbf{k} \]
is the position vector of a particle moving in space.

(a) [3 points] Find the velocity vector at \( t = 1 \).

(b) [3 points] Find the acceleration vector at \( t = 1 \).

(c) [2 points] What is the speed at \( t = 1 \)?
(d) [4 points] What is the angle between the position vector and the velocity vector at $t = 1$?
Hand-Graded Question [10 points]

15. Let $\mathbf{u} = \langle 0, 4, 1 \rangle$ and $\mathbf{v} = \langle 3, -2, 1 \rangle$.

(a) [5 points] Find the vector projection of $\mathbf{u}$ onto $\mathbf{v}$.

(b) [5 points] Write $\mathbf{u}$ as the sum of two vectors: one perpendicular to $\mathbf{v}$ and one parallel to $\mathbf{v}$.