1. Let

\[ A = \{(x, y, z)|x^2 + 7y^2 \leq 1\} \subset \mathbb{R}^3 \]

Which of the following statements are true?

I. \( A \) is closed
II. \( A \) is open
III. \( A \) is bounded
IV. \( A \) is connected

(a) I and II only
(b) I and III only
(c) I and IV only
(d) II and III only
(e) II and IV only
(f) I, II and III only
(g) I, II and IV only
(h) I, III and IV only
(i) II, III and IV only
(j) All are true
(k) None are true
2. Compute the maximal rate of change of \( f(x, y, z) = y^2 \cos z \sin x \) at the point \((\pi, \pi, \pi)\).

(a) \(-\pi^2\)
(b) \(-2\pi\)
(c) \(-\pi\)
(d) 0
(e) \(\pi\)
(f) \(2\pi\)
(g) \(\pi^2\)

3. Suppose you know

\[
\begin{align*}
  f(-2, 3) &= 7 \\
  f_x(-2, 3) &= 5 \\
  f_y(-2, 3) &= 1
\end{align*}
\]

At the point \((-2, 3)\), at what rate does the function increase in the direction of the vector \((1, -3)\)?

(a) 0
(b) \(\frac{1}{\sqrt{10}}\)
(c) \(\frac{2}{\sqrt{10}}\)
(d) \(\frac{3}{\sqrt{10}}\)
(e) \(\frac{4}{\sqrt{10}}\)
(f) \(\frac{5}{\sqrt{10}}\)
(g) \(\frac{6}{\sqrt{10}}\)
(h) \(\frac{7}{\sqrt{10}}\)
(i) \(\frac{8}{\sqrt{10}}\)
(j) \(\frac{9}{\sqrt{10}}\)
(k) \(\frac{10}{\sqrt{10}}\)
4. Let $f(x, y) = (3x - y)^2$.

Use a linear approximation at the point $(1, 2)$ to approximate $f(2, 3)$.

(a) 3  
(b) 4  
(c) 5  
(d) 6  
(e) 7  
(f) 8  
(g) 9  
(h) 10  
(i) 11  
(j) 12  
(k) 13

5. Consider the surface defined by the equation

$$x^2z + 3yz^2 + 3xyz = 7$$

The plane that is tangent to this surface at the point $(1, 1, 1)$ can be written in the form

$$5x + By + Cz = D$$

What is $B + C + D$?

(a) 31  
(b) 32  
(c) 33  
(d) 34  
(e) 35  
(f) 36  
(g) 37  
(h) 38  
(i) 39  
(j) 40
6. Let

\[ f(x, y) = xy^2 + x^5y^4 - \frac{x}{y^2} \]

Let \( T_2(x, y) \) be the second order Taylor polynomial centered at the point \((1, 1)\).

\[ T_2(1, 1) = \]

(a) 0 
(b) 1 
(c) 2 
(d) 3 
(e) 4 
(f) 5 
(g) 6 
(h) 7 
(i) 8 
(j) 9 
(k) 10 

7. Let

\[ f(x, y) = x^3y \]

Let \( T_2(x, y) \) be the second order Taylor polynomial centered at the point \((1, 2)\).

\[ T_2(0, 2) = \]

(a) 0 
(b) 1 
(c) 2 
(d) 3 
(e) 4 
(f) 5 
(g) 6 
(h) 7 
(i) 8 
(j) 9 
(k) 10
8. Suppose \( f(x, y) \) attains a maximum value of 17 at the point \((3, 4)\).
Which of the following is guaranteed to be a tangent plane to the surface?

(a) \( x = 3 \)
(b) \( y = 4 \)
(c) \( z = 17 \)
(d) \( z = (x - 3) + (y - 4) + 17 \)
(e) \( (x - 3) + (y - 4) + 17z = 0 \)
(f) None of the above are guaranteed to be tangent to the surface.

9. Which of the following statements about a smooth function \( f(x, y) \) must always be true?

I. If the tangent plane to \( f \) at a point \( p \) is parallel to the plane \( z = 0 \), then \( f \) has either a local maximum or minimum at the point \( p \).

II. If \( U \) is an open subset of the plane, and \( f \) has no critical points in \( U \), then \( f \) has no maximum or minimum in \( U \).

III. If \( C \) is a closed subset of the plane, and \( f \) has no critical points on \( C \), then \( f \) attains a maximum or minimum on the boundary of \( C \).

(a) I only
(b) II only
(c) III only
(d) I and II only
(e) I and III only
(f) II and III only
(g) I, II, and III only
(h) None must be true
10. Suppose you have a function $g(x, y)$ and you know the following
\begin{align*}
g(3, 7) &= 12 \\
g_x(3, 7) &= 0 \\
g_y(3, 7) &= 0 \\
g_{xx}(3, 7) &= -8 \\
g_{xy}(3, 7) &= -6 \\
g_{yy}(3, 7) &= -6
\end{align*}

Which must be true
(a) $g$ has a local maximum at $(3, 7)$
(b) $g$ has a local minimum at $(3, 7)$
(c) $g$ has a saddle point at $(3, 7)$
(d) The point $(3, 7)$ is neither a local max, local min nor saddle.
(e) More than one of the above are true
(f) None of the above are true

11. Let
\[ f(x, y) = y^4 - y^2 + x^2 - 2xy \]

The function $f$ has how many local maxima, local minima, and saddle points?
(a) 1 max, 2 min, 0 saddle
(b) 2 max, 1 min, 0 saddle
(c) 1 max, 1 min, 1 saddle
(d) 0 max, 2 min, 1 saddle
(e) 2 max, 0 min, 1 saddle
(f) 2 max, 2 min, 1 saddle
(g) 0 max, 4 min, 1 saddle
(h) 4 max, 0 min, 1 saddle
12. Find the minimal distance from the origin to the curve $3x^2 + 2xy + 3y^2 = 1$.

(a) 0
(b) $\frac{1}{2\sqrt{2}}$
(c) $\frac{1}{4}$
(d) $\frac{1}{2}$
(e) $\frac{1}{\sqrt{2}}$
(f) 1
(g) $\sqrt{2}$

13. Let $F$ be the vector field.

$$F = (2x \cos y, e^y - x^2 \sin y)$$

Let $\phi$ be a potential function such that $\phi(0, 0) = 1$. Find $\phi(1, 0)$.

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4
(f) 5
(g) 6
(h) 7
(i) 8
(j) 9
(k) The vector field $F$ has no potential function.
14. Let $F$ be the vector field.

\[ F = (3x^2y, x^3 + 2yz, y^2 + x) \]

Let $\phi$ be a potential function such that $\phi(0,0) = 1$. Find $\phi(1,1)$.

(a) 0 
(b) 1 
(c) 2 
(d) 3 
(e) 4 
(f) 5 
(g) 6 
(h) 7 
(i) 8 
(j) 9 
(k) The vector field $F$ has no potential function.

15. Let $F$ be the vector field

\[ F = \left( -\frac{y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right) \]

Which of the following are true?

I. $F$ has a potential function defined on the entire plane, $\mathbb{R}^2$.

II. $F$ has a potential function defined on the right half plane, $x > 0$.

III. $F$ has a potential function defined on the punctured plane, $(x,y) \neq (0,0)$.

IV. $F$ has no potential function.

(a) I only 
(b) II only 
(c) III only 
(d) IV only 
(e) I and II only 
(f) I and III only 
(g) II and III only 
(h) I, II and III only 
(i) All are true 
(j) None are true
16. Find the maximum and minimum of the function \( f(x, y) = xy \) on the domain \( 0 \leq y \leq 1 - x^2 \)? Be sure to state where the maximum and minimum occur.
17. Let

\[ f(x, y) = xy^2 - \frac{1}{2} y^4 - \frac{1}{3} x^3 \]

(a) Find all critical points of \( f \).

(b) For each critical point of \( f \), determine everything the second derivative test tells you about that critical point.