

This examination has twenty problems, of which the first fifteen are modifications of the recommended homework problems. The remaining five problems are in the same spirit. Each problem will be graded on a three-point scale, with 3=fully correct, 2=mostly correct, 1=mostly incorrect, and 0=fully incorrect. Please draw a box or oval around each final answer so I can locate it easily. Of course, for me to assign partial credit, I will also need to see your steps. This is especially important when you use the statistical capabilities of your calculator. For example, if you use the `normalcdf` function to solve a problem, one step should be to write down the function and its arguments *exactly as they appear* on your calculator screen (except you don't need to wrap the text the way it appears on the small calculator screen).

1. Find the middle three numbers in the eleventh row of Pascal's triangle.

2. A mathematics contest is held among three high schools. Each school enters a team of four students. The twelve contestants are ranked from 1 (best performance) to 12 (worst performance). The team that has the overall best performance (i.e., the lowest sum of ranks of the four students in the team) gets an award. In how many ways can the 12 ranks be assigned among the three teams without distinguishing between the individual ranks of the students in each team (since only the sum of their ranks matters)?

3. Let  $A$  and  $B$  be two events. Suppose that  $P(A) = p$ ,  $P(B) = 0.3$ , and  $P(A \cup B) = 0.8$ . For what value of  $p$  will  $A$  and  $B$  be independent?

4. Two fair dice are rolled. Let  $X$  be the absolute difference between the outcomes of the two dice. For example, if the outcomes are 2 and 6, then  $X = 4$ . Find  $P(2 \leq X \leq 4)$ .

5. A random variable  $X$  has p.d.f.  $f(x) = c(1+x)^{-3}$  for  $x \geq 0$  and  $f(x) = 0$  for all other values of  $x$ . Find the constant  $c$  so that  $f(x)$  is a p.d.f. Then find the c.d.f. of  $X$  and use it to determine the median of  $X$ .

6. Find the variance of a binomial random variable  $\text{Bin}(2,0.4)$  from its moment generating function. (Of course, feel free to check your work using the formula for variance of a binomial.)

7. An elderly patient is examined for heart and lung problems using scores  $X$  and  $Y$ , respectively. A score has a value of 0 if there is no impairment, 1 if there is mild impairment, and 2 if there is severe impairment. Suppose that the distributions of  $X$  and  $Y$  in the population are as follows:

$$P(X=0) = 0.7, \quad P(X=1) = 0.2, \quad P(X=2) = 0.1$$

$$P(Y=0) = 0.6, \quad P(Y=1) = 0.2, \quad P(Y=2) = 0.2$$

Let  $T = X + Y$  be the total score for lung and heart impairment for a patient. Find  $\text{Var}(T)$  if  $X$  and  $Y$  are independent of each other.

8. In a shotgun seminar, the topic is announced one week in advance, but the speaker is randomly chosen when the seminar begins. Suppose you attend twelve seminars with a total of eight participants (yourself included). Suppose the eight participants are divided into four teams of two for preparation. What is the probability that your team (i.e., you or your partner) will be selected twice?

9. An office supply warehouse receives an order for four computers. The warehouse has fifty computers in stock, of which six are defective. The order is filled by randomly drawing from the computers in stock. What is the probability that the customer receives exactly one defective computer in the shipment of four?

10. A typist makes a typographical error at the rate of 1 every 13 pages. Let  $X$  be the number of errors in a manuscript of 150 pages. Assuming that  $X$  has a Poisson distribution, calculate the probability that the manuscript has at least 10 errors.

11. The final series between two leagues has five games, and the team that wins three games first wins the series. Suppose that Team A has probability 0.6 of winning each game, while Team B has probability 0.4 of winning each game, and that the outcomes of the games are mutually independent. Find the probability that Team A wins the series.

12. Let  $X$  = the time between jobs arriving at a workstation. Assume that  $X$  is exponential with a mean of 15 minutes. What is the probability that no jobs arrive in a 20-minute interval?

13. Suppose that the monthly phone bills of a population of households are normally distributed with a mean of \$110 and standard deviation of \$30. A random sample of 29 households is taken from this population. What dollar amount will the average phone bill exceed with probability 20%?
14. To assess the effect of smoking during pregnancy on premature delivery, mothers of preterm infants are matched by age and number of previous pregnancies to mothers of full term infants. Both are asked about their smoking habits during pregnancy. (i) Tell whether the study is a survey, a prospective study, or a retrospective study. (ii) Tell whether the study is comparative or descriptive.

15. A company employs 1000 men and 700 women, of whom 100 men and 50 women are in management positions. A survey is planned to study employee attitudes towards company options on health care. Attitudes appear to differ by gender and by management/regular employee status. Describe a sampling plan that would provide attitude information by each subgroup defined by gender and management status as well as companywide, based on a total sample of 400 people.

16. Let  $X$  = the time between jobs arriving at a workstation. Assume that  $X$  is exponential with a mean of 15 minutes. What is the probability that exactly two jobs arrive in a 20-minute interval?



17. The ACT is said to have a mean of 18 and a standard deviation of 6. Assume that the reported scores reported are rounded to the nearest whole number from an underlying normal distribution  $N(18,6^2)$  (or to 0 or 36 in the case the underlying scores lie outside the 0-36 range). Find the probability a randomly chosen student will have a reported score of 25 or more. Also find the probability a randomly chosen student will have a reported score of exactly 25.

18. Given the information in Problem 17, determine the values of the underlying normal  $N(18,6^2)$  between which the middle 50% of all examinees lie. (That is, determine the distance between the 25<sup>th</sup> and 75<sup>th</sup> percentiles.)

19. The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35%. Of those coming to a PCP's office, 30% are referred to specialists and 40% require lab work. Determine the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

20. The lifetime of a printer costing \$200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, and a one-half refund if it fails during the second year. If the manufacturer sells 100 printers, how much should it expect to pay in refunds?