Math 416, Fall 2013

Assignment 5, 3 October 2013

PROGRESS: We are finishing Chapter 7 and will begin Chapter 8 next week.
DUE: This assignment is due Thursday, 10 October at the start of class.

PROBLEMS:

1. Pg 104— : 1, 6 (The result from problem 5 is useful here.), 8, 17
2. State and prove the following two variations on Schwarz's Lemma;
   (a) Suppose f is holomorphic in the closed unit disk and $|f(z)| \leq 1$ on the boundary. If $f(i/2) = 0$ then $|f(-i/2)| \leq$??
   (b) With the same hypothesis on f; if $f(0) = f'(0) = 0$ then for all z in the disk $|f(z)| \leq |z|$.
3. Suppose that f is an entire function that is real on the real axis. Show that if f is an even function on the real axis; $f(x) = f(-x)$ for all real x then f is an even function; $f(z) = f(-z)$ for all $z \in \mathbb{C}$.
4. The function $f(z) = e^z$ is an entire periodic function with period $2\pi i$; for all z, $f(z + 2\pi i) = f(z)$. Suppose $g(z)$ is an entire function with two independent periods, 1 and i; that is, for all z, $g(z) = g(z+1) = g(z+i)$. Show g is constant.