1. Consider the function \( f(x) = \cos x^3 \).

(a) Calculate \( f''(x) \).

\[
\begin{align*}
  f'(x) &= -\sin x^3 \cdot 3x^2 = -3x^2 \sin x^3 \\
  f''(x) &= -6x \sin x^3 - 3x^2 \cos x^3 \cdot 3x^2 = -6x \sin x^3 - 9x^4 \cos x^3.
\end{align*}
\]

(b) Using your result from part (a), find an upper bound for \(|f''(x)|\) on the interval \([-3, 0]\).

We use the rules from class repeatedly:

\[
|f''(x)| = \left| -6x \sin x^3 - 3x^2 \cos x^3 \cdot 3x^2 \right| = \left| -6x \sin x^3 - 9x^4 \cos x^3 \right| \\
\leq \left| 6|x| \cdot |\sin x^3| + 9|x^4| \cdot |\cos x^3| \right|.
\]

Then \(|\cos x^3| \leq 1\), similarly \(|\sin x^3| \leq 1\). On the interval \([-3, 0]\), we have \(|x| \leq 3\) and \(|x^4| = |x|^4 \leq 3^4 = 81\). Thus, \(|f''(x)| \leq 6 \cdot 3 + 9 \cdot 81 = 747\).

**Note:** It is not correct to bound by \(\sin 27\), as \(|\sin|\) takes on higher values on the given interval.

(c) Using your value of \(A\) from part (b), find a value of \(n\) so that \(T_n\) approximates \(\int_{-3}^{0} \cos x^3 \, dx\) with error of less than 0.05.

We want to find \(n\) so that

\[
\text{error } T_n \leq \frac{747 \cdot 3^3}{12 \cdot n^2} \leq 0.05.
\]

We solve:

\[
\frac{747 \cdot 3^3}{12 \cdot 0.05} \leq n^2, \quad \text{i.e.,} \quad \sqrt{\frac{747 \cdot 3^3}{12 \cdot 0.05}} \leq n.
\]

It is perfectly fine to leave your solution in this form on a quiz. If you have a calculator, you can find the square root to be 183.3, so
$n = 184$. **Note:** As I’ve discussed with some of you individually, on error estimations it’s perfectly acceptable to be sloppier with the bounds. For example, by taking the higher upper bound for $|f''(x)|$ of 800, you have easier numbers to calculate with:

$$
\sqrt{\frac{800 \cdot 3^3}{12 \cdot 0.05}} = \sqrt{200 \cdot 3^2 \cdot 20} = 60 \cdot \sqrt{10} < n,
$$

and (since $\sqrt{10} < 4$) we see that $n = 240$ will also give the desired error. Which is correct, 240 or 184? **Both** give error of less than 0.05, and they are close enough that it doesn’t make a great deal of difference for calculation time, so either is acceptable.