Math 128
Quiz 3 Solutions

Correct answers without work are worth no credit!

1. Consider the function $f(x) = \cos x^3$.

   (a) Calculate $f''(x)$.

$$f'(x) = -\sin x^3 \cdot 3x^2 = -3x^2 \sin x^3$$

$$f''(x) = -6x \sin x^3 - 3x^2 \cos x^3 \cdot 3x^2 = -6x \sin x^3 - 9x^4 \cos x^3.$$

   (b) Using your result from part (a), find an upper bound for $|f''(x)|$ on the interval $[-3, 0]$.

   We use the rules from class repeatedly:

$$\begin{aligned} |f''(x) &= |-6x \sin x^3 - 3x^2 \cos x^3 \cdot 3x^2 = -6x \sin x^3 - 9x^4 \cos x^3| \\ &\leq |6x \sin x^3| + |9x^4 \cos x^3| \\ &\leq 6|x| \cdot |\sin x^3| + 9|x^4| \cdot |\cos x^3|. \end{aligned}$$

   Then $|\cos x^3| \leq 1$, similarly $|\sin x^3| \leq 1$. On the interval $[-3, 0]$, we have $|x| \leq 3$ and $|x^4| = |x|^4 \leq 3^4 = 81$. Thus, $|f''(x)| \leq 6 \cdot 3 + 9 \cdot 81 = 747$.

   **Note:** It is *not* correct to bound by $\sin 27$, as $|\sin|$ takes on higher values on the given interval.

   (c) Using your value of $A$ from part (b), find a value of $n$ so that $T_n$ approximates $\int_{-3}^{0} \cos x^3 \, dx$ with error of less than 0.05.

   We want to find $n$ so that

$$\text{error } T_n \leq \frac{747 \cdot 3^3}{12 \cdot n^2} \leq 0.05.$$

   We solve:

$$\frac{747 \cdot 3^3}{12 \cdot 0.05} \leq n^2, \quad \text{i.e.,} \quad \sqrt{\frac{747 \cdot 3^3}{12 \cdot 0.05}} \leq n.$$

   It is perfectly fine to leave your solution in this form on a quiz. If you have a calculator, you can find the square root to be 183.3, so

$n = 184$. **Note:** As I've discussed with some of you individually,

on error estimations it's perfectly acceptable to be sloppier with the bounds. For example, by taking the higher upper bound for $|f''(x)|$ of 800, you have easier numbers to calculate with:

$$\sqrt{\frac{800 \cdot 3^3}{12 \cdot 0.05}} = \sqrt{200 \cdot 3^2 \cdot 20} = 60 \cdot \sqrt{10} < n,$$

and (since $\sqrt{10} < 4$) we see that $n = 240$ will also give the desired error. Which is correct, 240 or 184? *Both* give error of less than 0.05, and they are close enough that it doesn't make a great deal of difference for calculation time, so either is acceptable.