Math 493

Midterm Examination 2 – November 12, 2010

Name ______________________________

General Instructions: Please answer the following, showing all your work, and without the aid of books or notes.

1. (1 points each) True/False. Please read the statements carefully, as no partial credit will be given.

   (a) _____ If X, Y, and Z are discrete random variables such that X and Y are independent, and Y and Z are independent, then X and Z are also independent.

   (b) _____ If A and B are disjoint events, then $P(A \cap B) = P(A) \cdot P(B)$.

   (c) _____ If X, Y, and Z are continuous random variables such that X, Y, and Z are mutually independent, then X and Z are also independent.

   (d) _____ If A, B, and C are mutually independent events, then

   $$P(A \cap B^c) = P(A) \cdot P(B^c).$$

   (e) _____ If Z is a standard normal random variable, then the density function for $2Z$ is $\frac{2}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$.

   (f) _____ If Z is a standard normal random variable, then the density function for Z is an even function, therefore the conditional density function for Z given that $Z > 0$ is $\frac{2}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$.

   (g) _____ If X, and Y are independent continuous random variables with density function f_X and f_Y, then the joint density function $f_{X,Y}(x, y) = f_X(x) f_Y(y)$.

   (h) _____ If X and Y are independent discrete random variables, then the events $\{X = 2 \text{ or } 3\}$ and $\{Y = 1 \text{ or } 2\}$ are independent.

2. (a) (6 points) A rare disease called SIRS is known to occur in about 0.0002% of the population. Using the Poisson approximation, find the probability that there are ≤ 2 cases of SIRS in St. Louis County (population 1,000,000). You can leave your answer in terms of e.

(b) (1 point) Is the probability in part (a) greater than or less than 50%? Explain why or why not.

(That you can estimate this by hand illustrates the computational power of the Poisson approximation!)

(c) (3 points) Let $m_1(k) = b(n, \frac{1}{n}, k)$ be a binomial distribution, and $m_2(k)$ be the Poisson distribution with parameter $\lambda = 1$. Find the variation distance $\|m_1 - m_2\|$.

3. (a) (4 points) Let Z be an exponential random variable with parameter λ. Find the density function of $\sqrt{Z}$.

(b) (4 points) Without using the unproved fact that continuous images of independent random variables are independent, show that if X and Y are independent continuous random variables, then X and $-Y$ are also independent, as are $-X$ and $-Y$.

(c) (4 points) Show that if X and Y are independent continuous random variables, then

$$F_{X+Y}(a) = P(X + Y \leq a) = \int_{-\infty}^{\infty} F_X(a - y) \cdot f_Y(y)\, dy,$$

where F_X is the cumulative distribution function of X, and f_Y is the density function of Y.