

Math 131

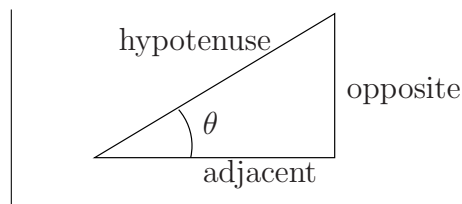
Trigonometry Overview – February 16, 2009

1. Definition

(a) Ratios in right triangles:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

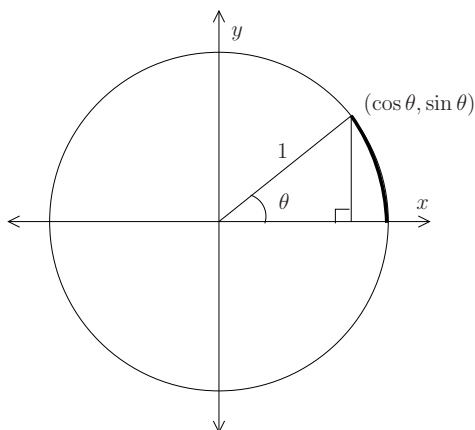
$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$



**Special case:** Hypotenuse = 1. (Here,  $\sin \theta = \text{opposite}$  and  $\cos \theta = \text{adjacent}$ .)

**A problem with this definition:** What is  $\sin \frac{3\pi}{2} = \sin 270^\circ$ ?

(b) Points on the circle with radius 1, which we call the *unit circle*:  
(Place a triangle)



**Note:** If  $\theta = \frac{\pi}{4}$ , then the length of the shaded part of the unit circle is  $\frac{\pi}{4}$ . (This is where radians come from!)

2. Useful values.

$\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	$\frac{7\pi}{6}$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1		
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0		

Use the symmetry in the circle above to fill in the blanks.

### 3. Identities

(a) Pythagorean Theorem:

$$\sin^2 \theta + \cos^2 \theta = 1.$$

(95% of the time, this is what you'll use.)

(b) Sums:

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

### 4. Derivatives and integrals and limits

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\frac{d}{dx} \sin x = \cos x \quad , \quad \frac{d}{dx} \cos x = -\sin x$$

$$\int \sin x = -\cos x + C \quad , \quad \int \cos x = \sin x + C$$

### 5. Other trig functions:

$$\tan x = \frac{\sin x}{\cos x}, \quad \sec x = \frac{1}{\cos x}, \quad \csc x = \frac{1}{\sin x}, \quad \cot x = \frac{1}{\tan x}$$

For most problems, you should start by expressing any other trig functions in terms of sin and cos.

### 6. Graphs. (Note that here, as always, sin and cos are in radians.)

