MAIN TOPICS FOR EXAM 1

(1) Fundamental definitions
  (a) Group
  (b) Subgroup
  (c) Homomorphism (Iso/auto-morphism)
  (d) Normal subgroup and conjugation
  (e) Examples: S_n, D_n, C_n, matrix groups, etc
(2) Fundamental lemmas
  (a) Uniqueness of identity and inverse
  (b) Cancellation
  (c) Characterizations of normal subgroup
(3) Fundamental theorems
  (a) Lagrange's Theorem
  (b) Isomorphism Theorem
  (c) Correspondence Theorem
  (d) 2nd Isomorphism Theorem
  (e) Diamond Theorem
(4) Group actions
  (a) Definition
  (b) Examples: conjugation, permutations, automorphisms, right multiplication
  (c) Characterization: a map into $\operatorname{Sym} X$
  (d) Stabilizers and kernels – faithful actions
  (e) Orbits – transitive actions
  (f) "Isomorphism Theorem" for actions: $\mathcal{O} \leftrightarrow \{Hg\}$
  (g) Finite Counting Principle
  (h) Stabilizers of interesting actions: $Z(G)$, $C_G(x)$, $N_G(H)$, etc.
(5) Applications of group actions
  (a) Cayley's Theorem
  (b) Product counting $|HK| = |H| \cdot |K|/|H \cap K|$
  (c) Finding normal subgroups
  (d) Counting conjugacy classes of elements/subgroups
  (e) automorphisms and inner automorphisms
(6) Some nice results
  (a) p-groups have non-trivial center
  (b) Cauchy's theorem
    (a) and (b) come from using the Finite Counting Principle mod p
(7) The symmetric group
  (a) cycle notation and the disjoint cycle decomposition
  (b) conjugacy in S_n – "change of basis"
(8) Homework results
  (a) commutator subgroup and abelian quotients