

MAIN TOPICS FOR EXAM 2

1. GROUP THEORY

- (1) Symmetric groups
 - (a) Decomposition into transpositions
 - (b) Sign of permutations
 - (c) Alternating groups
- (2) The Sylow theorems
 - (a) The E theorem – Sylow subgroups exists
 - (b) The D theorem – p -subgroups are contained in Sylow subgroups
 - (c) The C theorem – Sylow subgroups are conjugate
 - (d) Consequences of the C theorem: normal Sylow subgroups are characteristic
 - (e) Sylow counting
 - (f) Application to proving groups are not simple
- (3) Direct product of groups

2. RING THEORY

- (1) Ideals and quotients
 - (a) Isomorphism theorems
- (2) Rings and fields
 - (a) Quotient by a maximal ideal is a field
 - (b) Maximal ideals exist in all rings
 - (c) (Zorn's Lemma)
 - (d) Fraction fields
- (3) Euclidean domains, PIDs, and UFDs
 - (a) Examples of Euclidean domains
 - (b) Euclidean domains are PIDs and UFDs
 - (c) Gaussian integers
 - (d) Polynomial ring over a UFD is a UFD
- (4) Polynomial rings
 - (a) Degree
 - (b) Irreducible polynomials
 - (c) $\mathbb{Z}[x]$ and $F[x_1, x_2]$ are not PIDs
 - (d) Eisenstein Criterion
 - (e) Face rings
- (5) Field extensions