MAIN TOPICS FOR FINAL EXAM

All topics from Group Theory and Ring Theory (as covered in Topics for Exams 1 and 2), plus:

Field theory

(1) Field extensions
  (a) Dimension of K as F-algebra -> degree $[K : F]$.
  (b) If $K \supset L \supset F$ then $[K : F] = [K : L][L : F]$.
  (c) Algebraic vs trancendental elements.
  (d) Characterization of algebraic elements. (finite degree)
  (e) Minimal polynomial of an algebraic element.
  (f) Formal differentiation (definition only).
(2) Splitting fields
  (a) Roots of polynomials.
  (b) Construction of a field with a root of a polynomial -> $F[x]/(f)$.
  (c) The existence of splitting fields.
  (d) Uniqueness of splitting fields up to isomorphism fixing F.
  (e) Isomorphism $F(a) \cong F(b)$ if a, b are roots of $f \in F[x]$.
(3) Ruler and compass constructions
  (a) The set $\mathbb{K}$ of constructible numbers is a field with $\mathbb{Q} \subset \mathbb{K} \subset \mathbb{R}$.
  (b) $a \in \mathbb{K}$ iff there are $\lambda_1, \ldots, \lambda_n$ with $[\mathbb{Q}(\lambda_1, \ldots, \lambda_i) : \mathbb{Q}(\lambda_1, \ldots, \lambda_{i-1})] = 2$ for $i = 1, \ldots, k$.
  (c) Corollary: if a is constructible, then it lies in an extension $K/\mathbb{Q}$ with $[K : \mathbb{Q}] = 2^n$ for some n.
  (d) Applications: trisecting angle, doubling cube, etc.
(4) Galois theory
  (a) Definition of a Galois group
  (b) Galois correspondence between subgroup lattice and intermediate field lattice.
  (c) Galois field extensions.
    (i) Examples and anti-examples.
  (d) Action of a Galois group on roots of a polynomial.
(5) Sketch of further Galois theory
  (a) Equivalent conditions for a Galois extension.
  (b) If K/F is Galois, then the correspondence between subgroup lattice and intermediate field lattice is a bijection.