Math 494

Midterm Examination 1 – due February 23, 2011

Name ________________________________

**General Instructions:** Please answer the following, showing all your work and writing neatly. You may use calculators or computer algebra systems; and refer to your notes, to the textbooks Hogg-McKean-Craig and Grinstead-Snell (but not other books), or to Wikipedia (but not other websites, including Google). You may not discuss the exam problems with anyone other than Russ.

1. (1 points each) True/False. Please read the statements carefully, as no partial credit will be given.

   (a) _____ If $X$ has mgf $M(t) = e^{t^2/2}$, then $X$ is necessarily standard normal.

   (b) _____ If $X$ has distribution $N(0, \sigma^2)$, then $X^2$ has a Gamma distribution.

   (c) _____ If $X$ has distribution $\Gamma(\alpha, \beta)$, then $\frac{2X}{\beta}$ has a chi-squared distribution.

   (d) _____ If $X_1, \ldots, X_n$ are sampled from a Student $t$-distribution, then the expected value of the sample median is 0.

   (e) _____ If $(1, 2)$ is a 95% confidence interval for $\mu$, then there is a 95% probability that $\mu$ is between 1 and 2.

   (f) _____ If $X_1, \ldots, X_n$ are sampled from an $N(\mu, \sigma^2)$ distribution, then

   $$E((\overline{X} + 1) \cdot S^2) = \mu\sigma^2 + \sigma^2.$$

   (g) _____ If $X_1, \ldots, X_n$ are sampled from an exponentially distributed random variable $X$, and $M$ is the largest sample, then $\frac{n+1}{n} \cdot M - 1$ is an unbiased estimator for the maximum value of $X$.

   (h) _____ If $X$ and $Y$ are Gamma random variables, then $eX + \pi Y$ is a Gamma random variable.

2. You are examining an unknown radioactive material. The time $X$ between radioactive emissions is an exponential random variable, with unknown expected value $\mu$.

   (a) (4 points) Let $M$ be the median of 3 iid samples $X_1, X_2, X_3$ taken from the distribution of $X$. Find $E(M)$.

   (b) (5 points) You take 3 iid samples $x_1, x_2, x_3$ from $X$ and find the sample mean $\overline{x}$. Find a confidence interval for $\mu$ in terms of $\bar{x}$ and/or the probabilities from underlying distributions.

   (c) (1 point) Explain in 1-2 sentences the significance of the lower case notation $x_1, x_2, x_3$ in part (b) versus the upper case $X_1, X_2, X_3$ in part (a).

   (d) (2 points) Let $U$ be the number of radioactive emissions occurring in 10 units of time. (Recall from Math 493 that this is a Poisson random variable with parameter $\lambda = \frac{10}{\mu}$). You plan to take 3 iid samples $U_1, U_2, U_3$ from $U$. Describe two statistics on $U_1, U_2, U_3$ that give unbiased estimators for $\frac{1}{\mu}$.

3. (a) (4 points) Prove or disprove: if $Y = Z + X$, where $Y$ has an $N(\mu, \sigma^2)$ distribution and $Z$ is standard normal, then $X$ is necessarily normal.

   (b) (3 points) Prove or disprove: there is a continuous real-valued variable $X$ with mgf $M_X(t) = \cos t$.

   (c) (3 points) Prove or disprove: there is a continuous real-valued variable $Y$ with mgf $M_Y(t) = \frac{1}{2}(1 + e^t)$.